The effect of curvature on quasi-modes in coronal loops

Tom Van Doorsselaere, Arnold Debosscher, Jesse Andries, Stefaan Poedts

tomvd@wis.kuleuven.ac.be

KU LEUVEN
Departement WISKUNDE

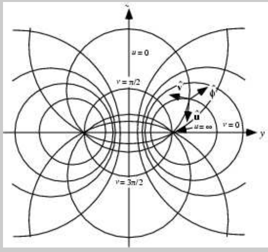
Celestijnenlaan 200 B,
B–3001 LEUVEN
BELGIUM

1. Abstract

We linearize the ideal MHD-equations to find the frequencies and damping rates of the ideal quasi-modes in coronal loops taking into account the curvature along the loop.

In the limit of small curvature, the quasi-mode frequencies remain unchanged, but, as a consequence of the curvature, quasi-mode oscillations are slightly more damped.

2. Toroidal coordinates (u, v, ϕ)



$$x = \frac{a \sinh u \cos\phi}{\cosh u - \cos v}$$
$$y = \frac{a \sinh u \sin\phi}{\cosh u - \cos v}$$
$$z = \frac{a \sin v}{\cosh u - \cos v}$$

3. Force-free magnetic field

$\vec{B} = B(u,v)\vec{e_\phi}$

\+

Force-free

\+

$\vec{\nabla}\cdot\vec{B} = 0$

$\longrightarrow$ $B(u,v) = B_{\mathrm{a}} \dfrac{(\cosh u - \cos v)}{\sinh u}$

(B_{a} is magnetic field on axis of loop)

4. Equilibrium & model & linearization

$Q(u,v,\phi,t) = \tilde{Q}(u,v)\exp i(k\phi - \omega t)$

MHD, $p = 0$, $g = 0$, static

$B(u,v)$ → EQUILIBRIUM → ρ free

$V_{\mathrm{A}}^2 = \dfrac{B^2}{\mu\rho}$

$$\rho(u,v) = \begin{cases} \rho_{\mathrm{a,e}} \frac{(\cosh u - \cos v)^4}{\sinh^4 u} & \text{for } u < u_0 \\ \rho_{\mathrm{a,i}} q(u) \frac{(\cosh u - \cos v)^4}{\sinh^4 u} & \text{for } u_0 \le u \le u_0 + d, \\ \rho_{\mathrm{a,i}} \frac{(\cosh u - \cos v)^4}{\sinh^4 u} & \text{for } u_0 + d < u \end{cases}$$

$$\omega_{\mathrm{A}}^2 = \begin{cases} \omega_{\mathrm{A,e}}^2 & \text{for } u < u_0 \\ \omega_{\mathrm{A,i}}^2 / q(u) & \text{for } u_0 \le u \le u_0 + d \\ \omega_{\mathrm{A,i}}^2 & \text{for } u_0 + d < u \end{cases}$$

Linearization

GOVERNING EQUATION:

$$\nabla^2 \left(\frac{b_\phi}{B}\right) = -\left(\frac{k^2\omega^2 (\cosh u - \cos v)^2}{\omega_A^2 a^2 \sinh^2 u}\right)\left(\frac{b_\phi}{B}\right)$$

u_0 is inner radius of loop
d is thickness of inhomogeneity

5. Solution of Governing equation

The general solution to the governing equation:

free phase!

$$\frac{b_\phi}{B} = \sum_\beta C_{e/i,\beta} \sqrt{\frac{\cosh u - \cos v}{\sinh u}} F_{e/i,\beta}(u) \exp\left(i\beta(v - v_0)\right)$$

β is integer (cfr. m in cylinder). $\alpha_{e/i} = \omega/\omega_{\mathrm{A,e/i}}$ and $F_{e/i,\beta}(u)$ equals

$$F_{\mathrm{i},\beta}(u) = \cosh^{-|\beta|} u \left(\coth^2 u\right)^{-\frac{1}{4}+\frac{k}{2}\sqrt{1-\alpha_{\mathrm{i}}^2}} \times$$
$${}_2F_1\left(\frac{1}{4} - \frac{k}{2}\sqrt{1-\alpha_{\mathrm{i}}^2} + \frac{|\beta|}{2}, \frac{3}{4} - \frac{k}{2}\sqrt{1-\alpha_{\mathrm{i}}^2} + \frac{|\beta|}{2}; 1+|\beta|; \frac{1}{\cosh^2 u}\right)$$
$$F_{\mathrm{e},\beta}(u) = \cosh^{-\beta} u \left(\coth^2 u\right)^{-\frac{1}{4}-\frac{k}{2}\sqrt{1-\alpha_{\mathrm{e}}^2}} \times$$
$${}_2F_1\left(\frac{1}{4} + \frac{k}{2}\sqrt{1-\alpha_{\mathrm{e}}^2} + \frac{\beta}{2}, \frac{3}{4} + \frac{k}{2}\sqrt{1-\alpha_{\mathrm{e}}^2} + \frac{\beta}{2}; 1+k\sqrt{1-\alpha_{\mathrm{e}}^2}; \tanh^2 u\right)$$

6. Connecting the external and internal region

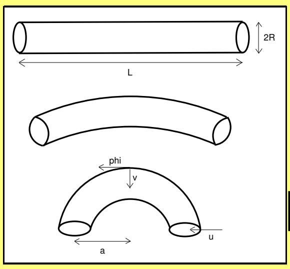
$$P_i - P_e = 0$$
$$\xi_i - \xi_e = \text{JUMP}$$

Equations for $C_{e/i,\beta}$

$$\frac{i\pi\omega_{\mathrm{A}}^2}{|\Delta|}(\alpha_{\mathrm{i}}^2 - 1)(\alpha_{\mathrm{e}}^2 - 1)C_{\mathrm{i},\beta}\left(\beta^2 + \frac{1}{2}\coth u_{\mathrm{A}} + \frac{3}{4}\frac{1}{\sinh u_{\mathrm{A}}}\right)$$
$$+ \frac{i\pi\omega_{\mathrm{A}}^2}{|\Delta|}(\alpha_{\mathrm{i}}^2 - 1)(\alpha_{\mathrm{e}}^2 - 1)\sum_{\delta\neq 0} C_{\mathrm{i},\beta+\delta}\frac{F_{\mathrm{i},\beta+\delta}(u_0)}{F_{\mathrm{i},\beta}(u_0)}\exp\left(-i\delta v_0\right)\frac{\exp\left(-|\delta| u_{\mathrm{A}}\right)}{2}\left(\coth u_{\mathrm{A}} + \frac{3}{4}(\delta+1)\frac{1}{\sinh u_{\mathrm{A}}}\right)$$
$$= C_{\mathrm{i},\beta}\left((\alpha_{\mathrm{i}}^2 - 1)\frac{d\log F_{\mathrm{e},\beta}}{du}(u_0) - (\alpha_{\mathrm{e}}^2 - 1)\frac{d\log F_{\mathrm{i},\beta}}{du}(u_0)\right.$$
$$\left. + \frac{1 - \coth u_0}{2}(\alpha_{\mathrm{i}}^2 - \alpha_{\mathrm{e}}^2)\right) + \sum_{\delta\neq 0} C_{\mathrm{i},\beta+\delta}\exp\left(-i\delta v_0\right)\frac{\exp\left(-|\delta| u_0\right)}{2}\frac{F_{\mathrm{i},\beta+\delta}(u_0)}{F_{\mathrm{i},\beta}(u_0)}(\alpha_{\mathrm{i}}^2 - \alpha_{\mathrm{e}}^2) \quad (*)$$

u_{A} is place of resonance
$u_0 < u_{\mathrm{A}} < u_0 + d$

7. Small curvature



Expand (*) for small curvature ε:

$$\varepsilon = \frac{R}{a} = \frac{\pi R}{Lk} = \frac{1}{\sinh u_0} \approx \frac{1}{\sinh u_{\mathrm{A}}}$$

No real part to first order on diagonal!

n-th off-diagonal $\propto \varepsilon^n$

8. Realistic coronal loops

Demand that (*) has a solution:

$$\omega = \omega_{0\mathrm{r}} - i\omega_{0\mathrm{i}}\left(1 + \varepsilon\frac{\frac{3}{4}}{\beta^2 + \frac{1}{2}}\right)$$

with

$$\omega_{0\mathrm{r}} = \frac{2\omega_{\mathrm{Ae}}^2\omega_{\mathrm{Ai}}^2}{\omega_{\mathrm{Ae}}^2 + \omega_{\mathrm{Ai}}^2},$$
$$\omega_{0\mathrm{i}} = -\frac{\frac{\pi\omega_{0\mathrm{r}}^2}{|\Delta|}\left(\frac{\omega_{0\mathrm{r}}^2}{\omega_{\mathrm{Ai}}^2} - 1\right)\left(\frac{\omega_{0\mathrm{r}}^2}{\omega_{\mathrm{Ae}}^2} - 1\right)\left(\beta^2 + \frac{1}{2}\right)}{2|\beta|\omega_{0\mathrm{r}}\left(\frac{\omega_{\mathrm{Ae}}^2 + \omega_{\mathrm{Ai}}^2}{\omega_{\mathrm{Ae}}^2\omega_{\mathrm{Ai}}^2}\right)}.$$

9. Conclusions

- No preferential oscillation direction
- No first-order change in quasi-mode frequency
- Quasi-modes slightly more damped
- Only coupling between neighbouring poloidal mode numbers