Kinetic Aspects of Coronal Heating

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- Collisional transport theory, basic assumptions
- Microphysics of heating by Coulomb collisions (viscosity, resistivity, conduction) and through dissipation of electric and magnetic wave energy
- Kinetic aspects of coronal heating related with Landau and cyclotron damping of plasma waves
- Kinetic result obtained by solving the Vlasov equation for particle velocity distributions in the solar corona

Kinetic Vlasov-Boltzmann theory

Detailed description of particle velocity distribution function in phase space:

$$\frac{df}{dt} + \mathbf{w}\cdot\frac{\partial f}{\partial \mathbf{x}} + (\mathbf{w}\times\mathbf{\Omega})\cdot\frac{\partial f}{\partial \mathbf{w}} + \left(-\frac{d}{dt}\mathbf{u} + \frac{q}{m}\mathbf{E}'\right)\cdot\frac{\partial f}{\partial \mathbf{w}} - \frac{\partial \mathbf{u}}{\partial \mathbf{x}} : \mathbf{w}\frac{\partial f}{\partial \mathbf{w}} = \mathcal{C}f$$

Coulomb collisions and/or **wave-particle interactions** are included via (acceleration ***A*** and diffusion *D*)

$$\mathcal{C}f = \frac{\partial}{\partial \mathbf{v}}\cdot\left(-\mathbf{A} + \mathcal{D}\cdot\frac{\partial}{\partial \mathbf{v}}\right)f$$

Relative velocity **w**, mean velocity **u**(**x**,t), gyrofrequency **Ω**, electric field **E'** in moving frame:

$$\mathbf{w} = \mathbf{v} - \mathbf{u}(\mathbf{x}, t),\ \ \mathbf{\Omega} = \frac{qB}{mc},\ \ \mathbf{E}' = \mathbf{E} + \frac{1}{c}\mathbf{u}\times\mathbf{B}$$

Convective derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\frac{\partial}{\partial \mathbf{x}}$$

Moments:
Velocity, pressure (stress) tensor, heat flux vector

$$< \mathbf{w} >= 0, \mathcal{P} = nm < \mathbf{w}\mathbf{w} >, \mathbf{Q} = nm < \mathbf{w}\frac{1}{2}w^2 >$$

$$\mathbf{\Pi} = \mathcal{P} - \mathcal{I}p \quad p = nk_BT = \frac{1}{3}Tr\mathcal{P}$$

Varying Coulomb collisions

Parameter	Chromo -sphere	Corona ($1R_S$)	Solar wind (1AU)
n_e (cm^{-3})	10^{10}	10^7	10
T_e (K)	10^3	1-2 10^6	10^5
λ (km)	10	1000	10^7

- Since N < 1, Coulomb collisions require kinetic treatment!
- Yet, only a few collisions (N ≥ 1) remove extreme anisotropies!
- Slow wind: N > 5 about 10%, N > 1 about 30-40% of the time.

Corona is weakly collisional, $\Omega_{i,e} >> \nu_{i,e}$, and strongly magnetized, $r_{i,e} << \lambda_{i,e}$

Classical transport theory of plasmas I

Assumption: Collisions allow only very small deviations from Maxwellians!

-> Particle velocity distributions can be expanded about a **local Maxwellian**.

$$F(\mathbf{w}, \mathbf{x}, t) = F_{\mathrm{M}}(n(\mathbf{x}, t), \mathbf{u}(\mathbf{x}, t), T(\mathbf{x}, t), \mathbf{w})$$

-> Chapman, Enskog, Cowling, and Grad polynomial expansions....

$$f(\mathbf{w}) = F_0(w) + \mathbf{w} \cdot \mathbf{F}_1(\mathbf{w})) + \mathbf{ww} : \mathcal{F}_2(\mathbf{w}) + ...$$

$$\mathbf{F}_1 \sim \mathcal{T}, \mathcal{N}, \mathcal{R}$$

$$\mathcal{F}_2 \sim \mathcal{U}$$

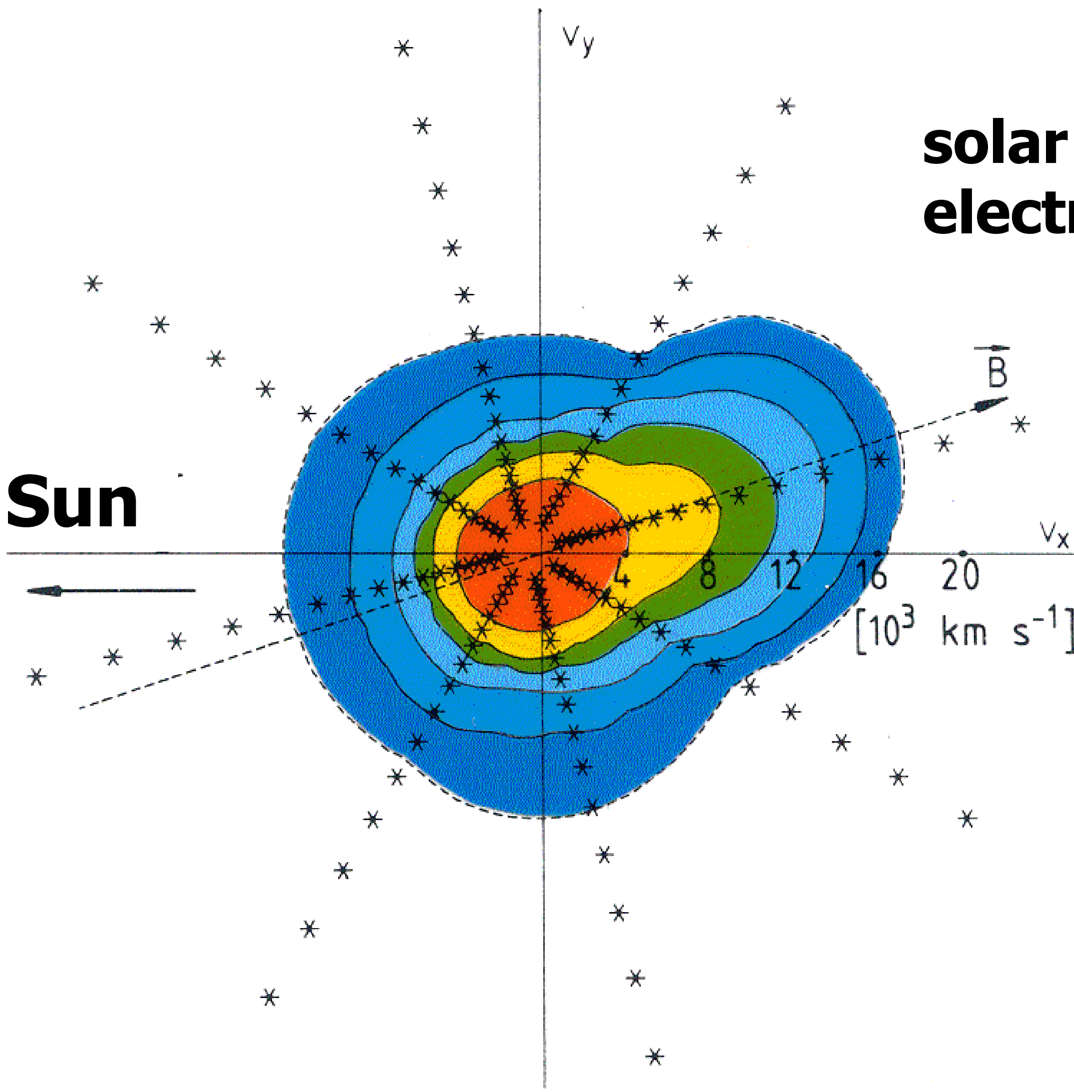
-> heat conduction, particle diffusion, electrical resistivity, viscosity

$$\mathcal{T} = \boldsymbol{\nabla} T(\mathbf{x}, t), \ \ \mathcal{N} = \boldsymbol{\nabla} n(\mathbf{x}, t), \ \ \mathcal{R} = m < \mathbf{v} \mathcal{C} f >$$

$$\mathcal{U} = \frac{1}{2}(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + [\frac{\partial \mathbf{u}}{\partial \mathbf{x}}]^T) - \frac{1}{3} \mathcal{I} \boldsymbol{\nabla} \cdot \mathbf{u}$$

Dum, 1990

Breakdown of classical transport theory



solar wind electrons

n_e = 3-10 cm^{-3}
T_e = 1-2 10^5 K
at 1 AU

- Strong heat flux tail
- Collisional free path λ_c much larger than temperature gradient scale L
- Polynomial expansion about a local Maxwellian badly converges, as $\lambda_c >> L$

Pilipp et al., JGR, **92**, 1075, 1987

Classical transport theory of plasmas II

Fluid theory: Collisional couplings appear via the momentum transfer rate *R*. Momentum equation of motion for each species separately:

$$nm\frac{d}{dt}\mathbf{u} = -\frac{\partial}{\partial \mathbf{x}} \cdot \mathcal{P} + nq\,[\mathbf{E} + \frac{1}{c}\mathbf{u} \times \mathbf{B}] + \mathcal{R}$$

← What is it?

Kinetic theory: Collisions included through perturbative solutions of the Boltzmann equation, in lowest order:

$$f(\mathbf{w}) = f^0 + \epsilon f^1 + \ldots.$$

$$\mathbf{w} \times \mathbf{\Omega} \cdot \frac{\partial f^0}{\partial \mathbf{w}} = \mathcal{C} f^0$$

Smallnes parameter: $\varepsilon \sim \lambda_c/L$

$$\mathcal{C} f \sim \frac{1}{\tau_c} f \quad (\frac{d}{dt})^{-1} \gg \tau_c \quad L \gg \lambda_c$$

Dum, Marsch, Pilipp, 1980

Solar wind electrons and ions strongly violate this assumption. Even expansions to higher order in **w** *(or cosine of pitch angle) do not converge!*

Functional form of the transport coefficients is wrong!

Kinetic processes in the solar corona and solar wind

• Plasma is multi-component and nonuniform

→ **multi-fluid or kinetic physics is required**

• Plasma is dilute and turbulent

→ **free energy for micro-instabilities**

→ **resonant wave-particle interactions**

→ **collisions by Fokker-Planck operator**

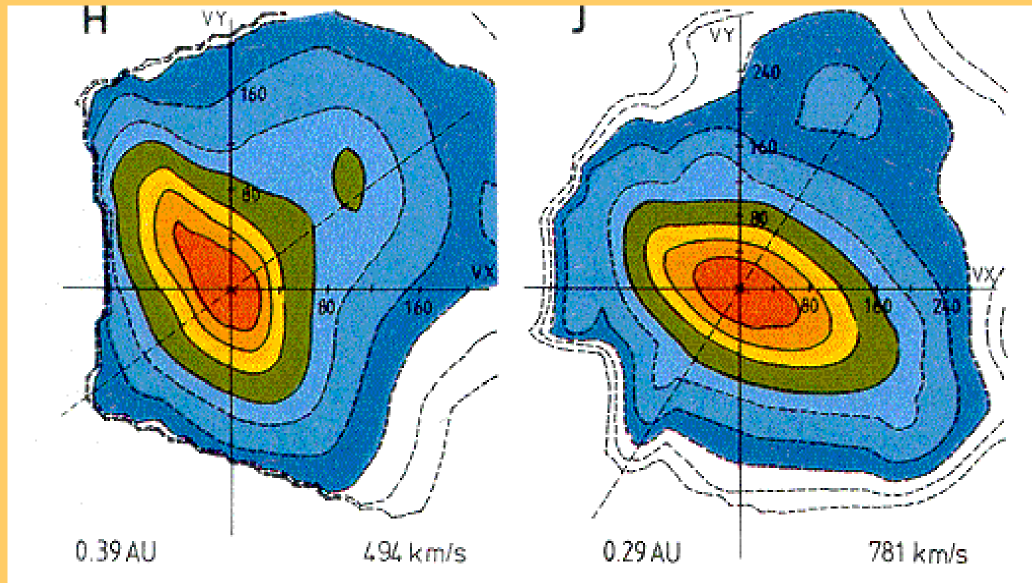
→ **scattering by waves, diffusion operator**

Problem: *Transport properties of the plasma, which involves multiple scales.....*

Protons and ion-cyclotron waves

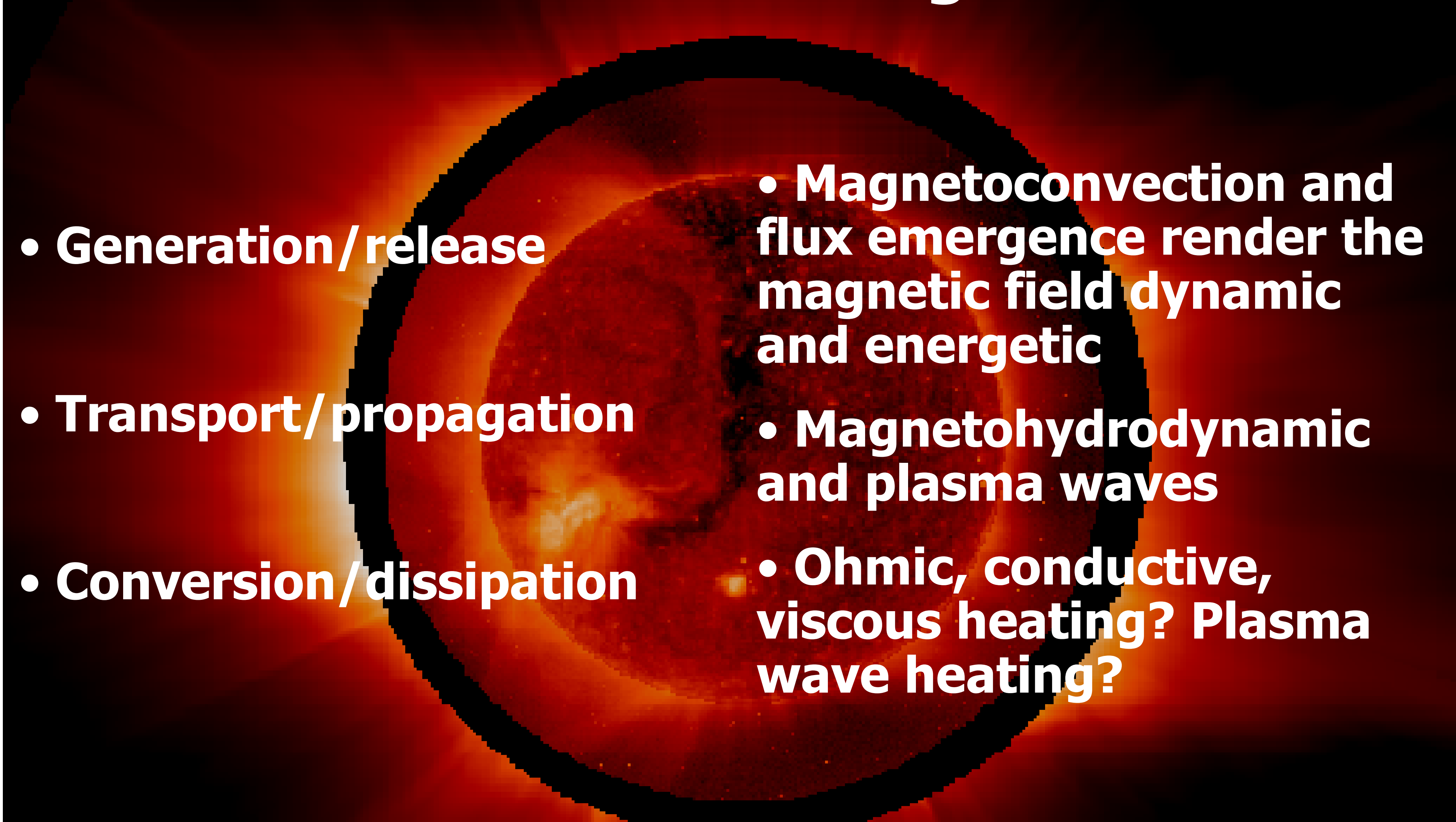
- Ion core temperature anisotropy (cyclotron resonance)
- Hot ion beam (coronal jets?)
- Electric current, e.g. due to ion-electron drift
- Loss-cone distribution
- Parametric decay of large-amplitude Alfvén waves
- Inhomogeneity, frequency sweeping, $\omega_{gi}(r)$.

Anisotropic isocontours of proton velocity distributions in fast solar wind (Helios)



Do these processes operate in the corona?

Coronal wave heating scenario



- Generation/release
- Transport/propagation
- Conversion/dissipation

- Magnetoconvection and flux emergence render the magnetic field dynamic and energetic
- Magnetohydrodynamic and plasma waves
- Ohmic, conductive, viscous heating? Plasma wave heating?

Microphysics of dissipation?

Collisional heating rates are too small

Chromosphere: $N = 10^{10}\ cm^{-3}$, $h_G = 400$ km

Perturbation scales: $\Delta L = 200$ km, $\Delta B = 1$ G, $\Delta V = 1$ km/s, $\Delta T = 1000$ K

Viscosity: $H_V = \eta\ (\Delta V/\Delta L)^2 = 2\ 10^{-8}$

Conductivity: $H_C = \kappa\ \Delta T/(\Delta L)^2 = 3\ 10^{-7}$

Resistivity: $H_J = j^2/\sigma = (c/4\pi)^2(\Delta B/\Delta L)^2/\sigma = 7\ 10^{-7}$

Radiative cooling: $C_R = N^2\Lambda(T) = 10^{-1}$ erg $cm^{-3}\ s^{-1}$

Smaller scale, $\Delta L \approx 200$ m, required!

$\lambda_{\mathbf{Coll}} \approx 1$ km

Wave-particle interactions

Dispersion relation using measured or model distribution functions $f(\underline{v})$, e.g. for electrostatic waves:

$$\varepsilon_L(\underline{k},\omega) = 0 \;\rightarrow\; \omega(\underline{k}) = \omega_r(\underline{k}) + i\gamma(\underline{k})$$

Dielectric constant is functional of $f(\underline{v})$, which may when being non-Maxwellian contain free energy for wave excitation.

$\gamma(\underline{k}) > 0 \rightarrow$ micro-instability......

Resonant particles:

$\omega(\underline{k}) - \underline{k} \cdot \underline{v} = 0$ (Landau resonance)

$\omega(\underline{k}) - \underline{k} \cdot \underline{v} = \pm \Omega_j$ (cyclotron resonance)

→ Energy and momentum exchange between waves and particles. Quasi-linear or non-linear relaxation.....

Heating and acceleration of ions by cyclotron and Landau resonance

$$\begin{pmatrix} \frac{\partial}{\partial t} U_{j\|} \\ \frac{\partial}{\partial t} V_{j\|}^2 \\ \frac{\partial}{\partial t} V_{j\perp}^2 \end{pmatrix}$$

acceleration

parallel heating

perpendicular heating

$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{B}_M(\mathbf{k}) \left(\frac{\Omega_j}{k}\right)^2 \frac{1}{1 - |\hat{\mathbf{k}} \cdot e_M(\mathbf{k})|^2}$$

$$\times \sum_{s=-\infty}^{+\infty} \mathcal{R}_j(\mathbf{k}, s) \begin{pmatrix} k_\| \\ 2k_\| w_j(\mathbf{k}, s) \\ s\Omega_j \end{pmatrix}$$

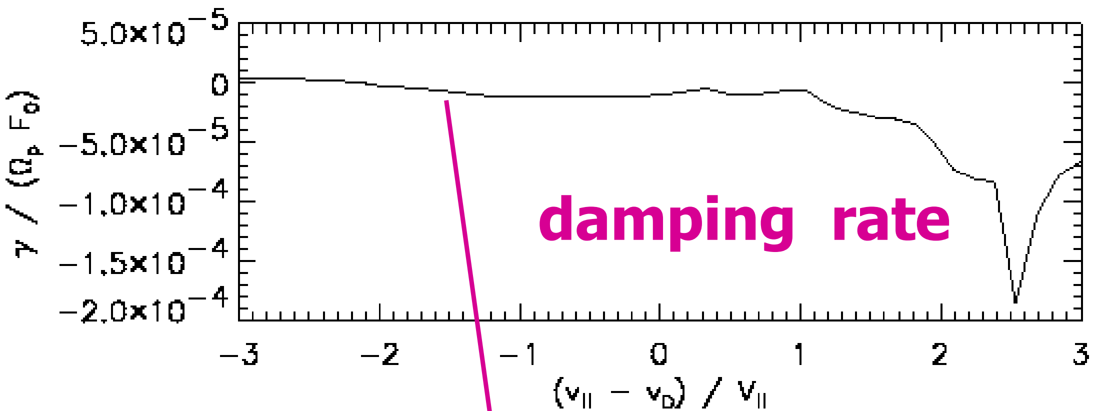
- *Wave spectrum and dispersion?*
- *Particle velocity distribution function?*

Marsch and Tu, JGR, **106**, 227, 2001

Ab initio kinetic calculations required!

Kinetics of coronal oxygen ions

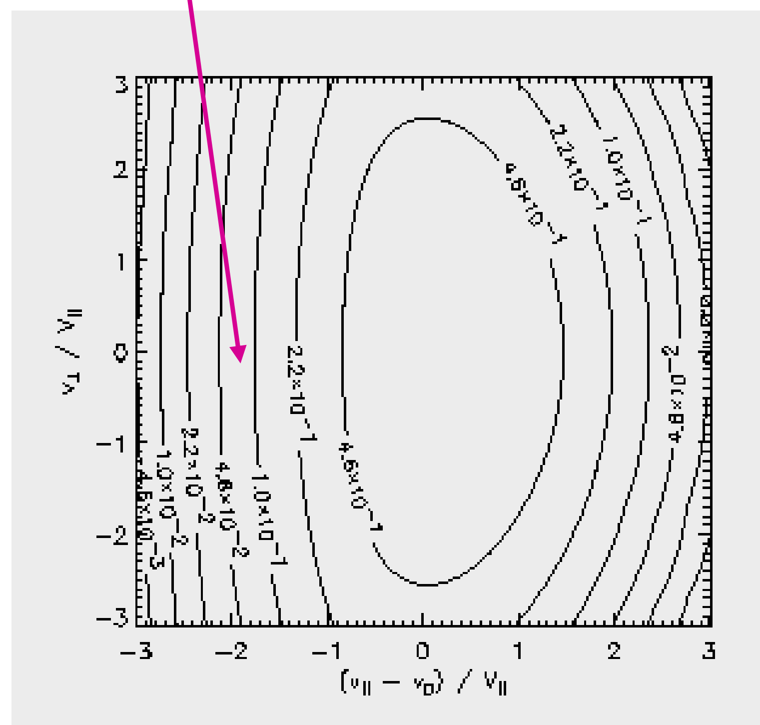
Plateau formation and marginal stability of the oxygen O^{5+} VDF at 1.44 R_s



Wave growth or damping vanishes for particles on the quasilinear plateau, where the pitch-angle gradient is zero in the wave frame.

$$f_j(w_\|, w_\perp) = \frac{F_{j\|}(w_\|)}{2\pi W_{j\perp}^2(w_\|)} \exp\left(-\frac{w_\perp^2}{2W_{j\perp}^2(w_\|)}\right)$$

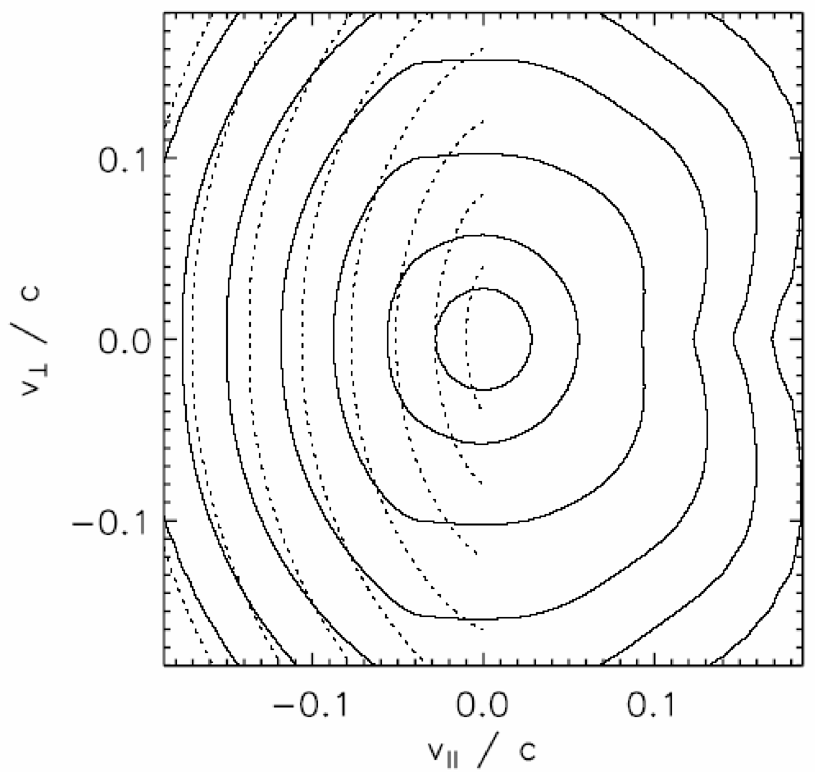
$$W_{j\perp}^2(w_\|) = \frac{F_{j\perp}(w_\|)}{F_{j\|}(w_\|)}$$



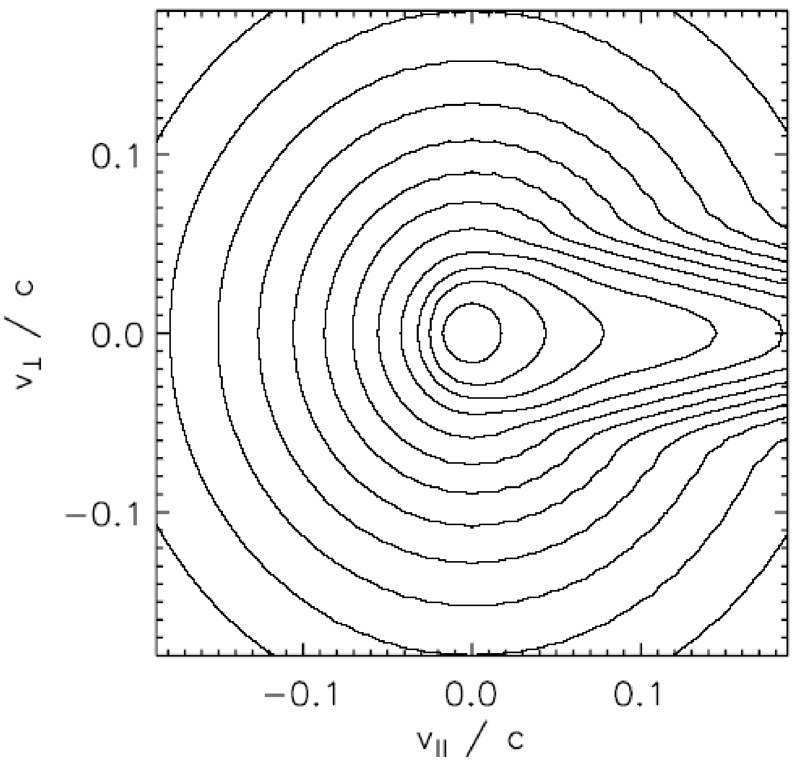
Vocks & Marsch, Ap. J. **568**, 1030 , 2002

Suprathermal coronal electrons caused by wave-particle interactions

s= 0.014 R_s



s= 6.5 R_s



Focusing
->
Strahl

Pitch-angle scattering
-> shell formation

Vocks and Mann, Ap. J., **593**, 1134, 2003

Kinetic physics of the solar corona – some conclusions

- Coronal imaging and spectroscopy indicate deviations of the plasma from thermal equilibrium

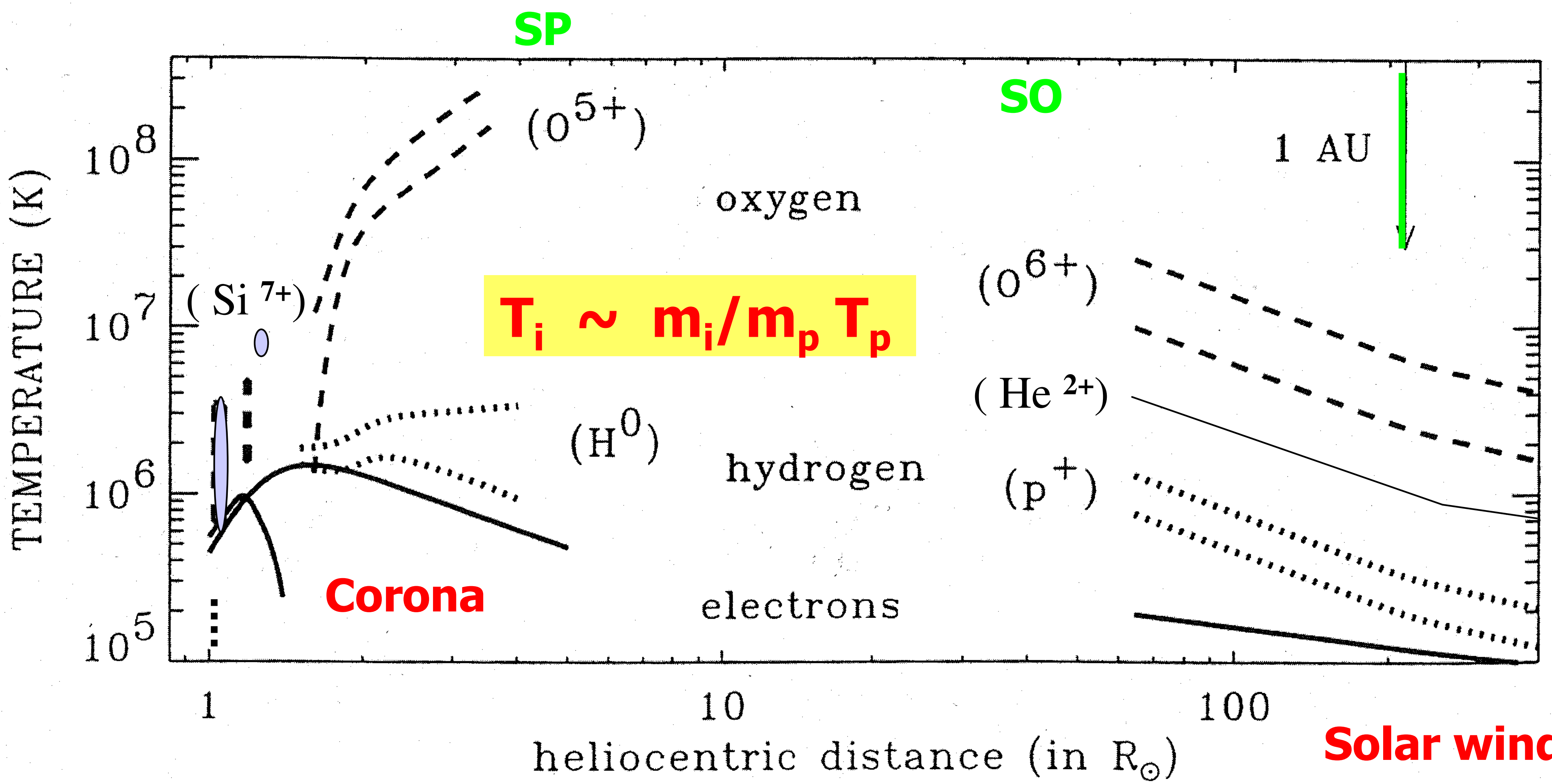
- Kinetic models allow one to understand essential observed features of the corona and solar wind

- Kinetic models with self-consistent wave spectra provide valuable physical insights in coronal heating

- The thermodynamics (heating) of the solar corona will ultimately require a fully kinetic plasma approach

- Turbulent wave transport as well as cascading and dissipation in the kinetic domain are not well understood

Solar corona and solar wind are far from thermal equilibrium



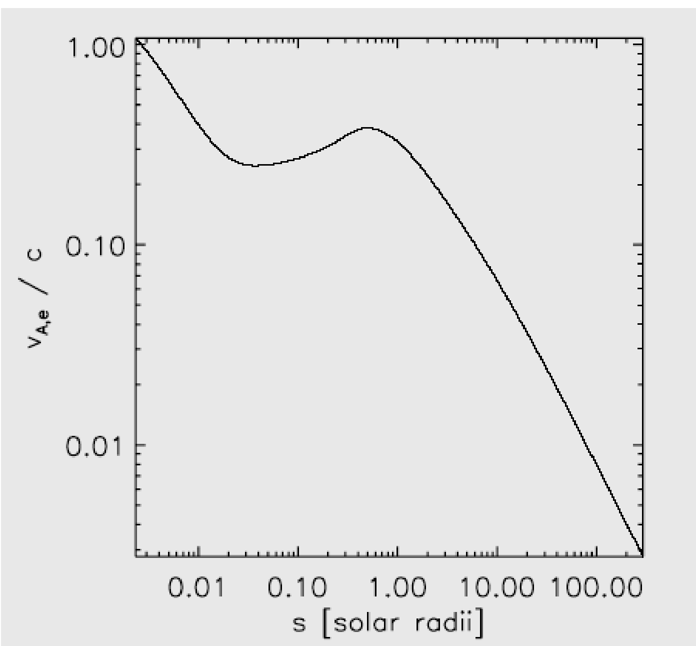
Cranmer et al., Ap.J., 2000; Marsch, 1991

Suprathermal coronal electrons caused by wave-particle interactions I

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial s} + \left(g_\parallel - \frac{e}{m_e} E_\parallel \right) \frac{\partial f}{\partial v_\parallel} + \frac{v_\perp}{2A} \frac{\partial A}{\partial s} \left(v_\perp \frac{\partial f}{\partial v_\parallel} - v_\parallel \frac{\partial f}{\partial v_\perp} \right)$$
$$= \left(\frac{\delta f}{\delta t} \right)_{w\text{-}p} + \left(\frac{\delta f}{\delta t} \right)_{\mathrm{Coul}} .$$

Boltzmann equation

A(s) flux tube area function



Electron pitch-angle scattering in the Whistler wave field

Phase speed $v_{A,e}$ in solar corona

Vocks and Mann, Ap. J., **593**, 1134, 2003

Semi-kinetic model of wave-ion interaction in the corona

$$F_{j\|}(w_\|) = 2\pi \int_0^\infty dw_\perp w_\perp f_j(w_\perp, w_\|)$$

$$F_{j\perp}(w_\|) = 2\pi \int_0^\infty dw_\perp w_\perp \frac{w_\perp^2}{2} f_j(w_\perp, w_\|)$$

$$\int_{-\infty}^{\infty} dw_\| F_{j\|}(w_\|) \begin{pmatrix} 1 \\ w_\| \\ w_\|^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ V_{j\|}^2 \end{pmatrix}$$

$$\int_{-\infty}^{\infty} dw_\| F_{j\perp}(w_\|) = V_{j\perp}^2$$

Reduced Velocity distributions

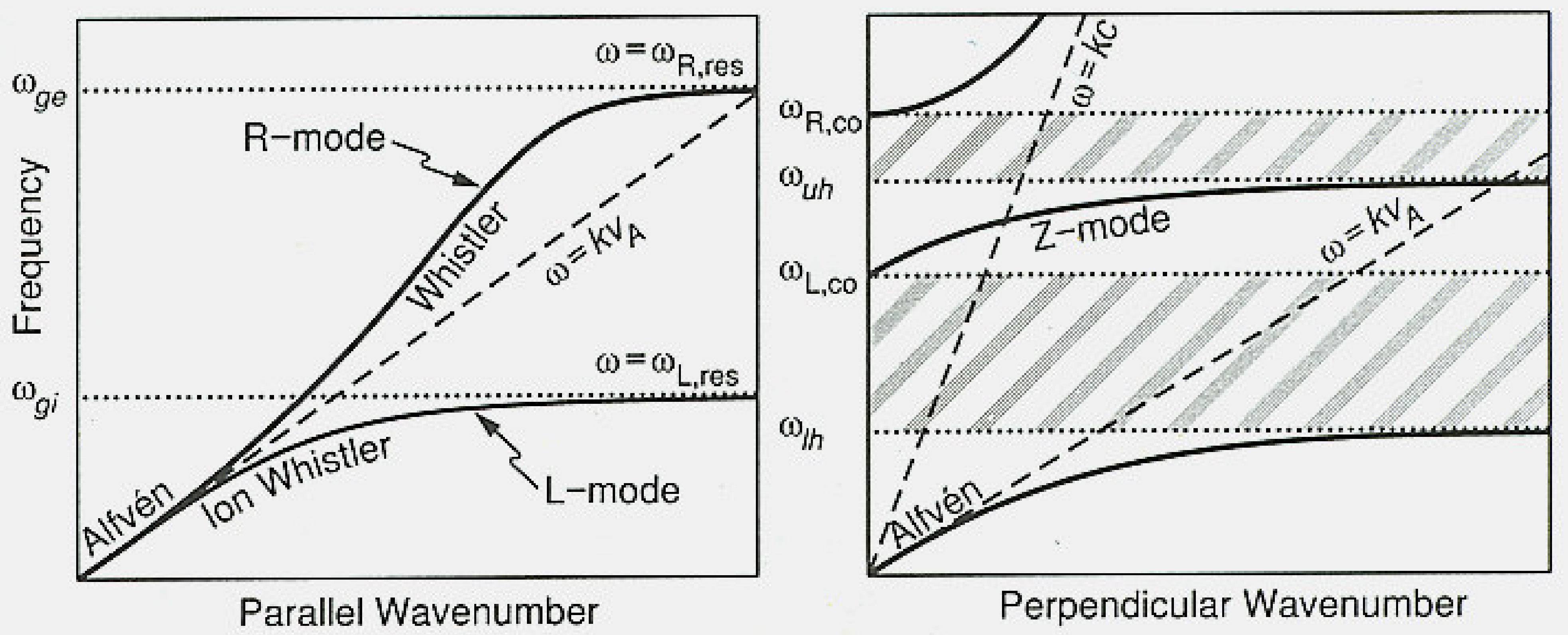
$$\frac{\partial F_\|}{\partial t} + v_\| \frac{\partial F_\|}{\partial s} + \left(\frac{q}{m}E_\| - g(s)\right)\frac{\partial F_\|}{\partial v_\|} + \frac{1}{2A(s)}\frac{\partial A(s)}{\partial s} \cdot 2\left(\frac{\partial F_\perp}{\partial v_\|} + v_\| F_\|\right) = \frac{\delta F_\|}{\delta t} + \frac{\delta F_\|}{\delta t}\Big|_{Coul.}$$

$$\frac{\partial F_\perp}{\partial t} + v_\| \frac{\partial F_\perp}{\partial s} + \left(\frac{q}{m}E_\| - g(s)\right)\frac{\partial F_\perp}{\partial v_\|} + \frac{1}{2A(s)}\frac{\partial A(s)}{\partial s} \cdot 4\left(v_{j\perp}^2\frac{\partial F_\perp}{\partial v_\|} + v_\| F_\perp\right) = \frac{\delta F_\perp}{\delta t} + \frac{\delta F_\perp}{\delta t}\Big|_{Coul}$$

Marsch, Nonlinear Proc. Geophys., **5,** 111, 1998

Vocks and Marsch, GRL, **28**, 1917, 2001

Coronal plasma waves



- **Ion cyclotron waves**, kinetic Alfvén waves, $\omega \leq \omega_{gi} = e_i B/m_i c$
- **Whistler waves**, $\omega \leq \omega_{ge} = eB/m_e c$

upper
lower
hybrid
waves

$$\omega_{uh}^2 = \omega_{ge}^2 + \omega_{pe}^2$$

$$\omega_{lh}^2 = \frac{\omega_{pi}^2 + \omega_{gi}^2}{1 + \omega_{pe}^2/\omega_{ge}^2}$$

Detectability of plasma waves

- **Spatial** (pixel size) and **temporal** (exposure/cadence) **resolution** be less than **wavelengths** and **periods** **-->** **presently not possible**
- **Spectral resolution** is sufficient to resolve Doppler shifts and broadenings, due to the integrated effects of the unresolved **high-frequeny turbulence** with a line-of-side **amplitude** ξ, which leads to an effective ion temperature:

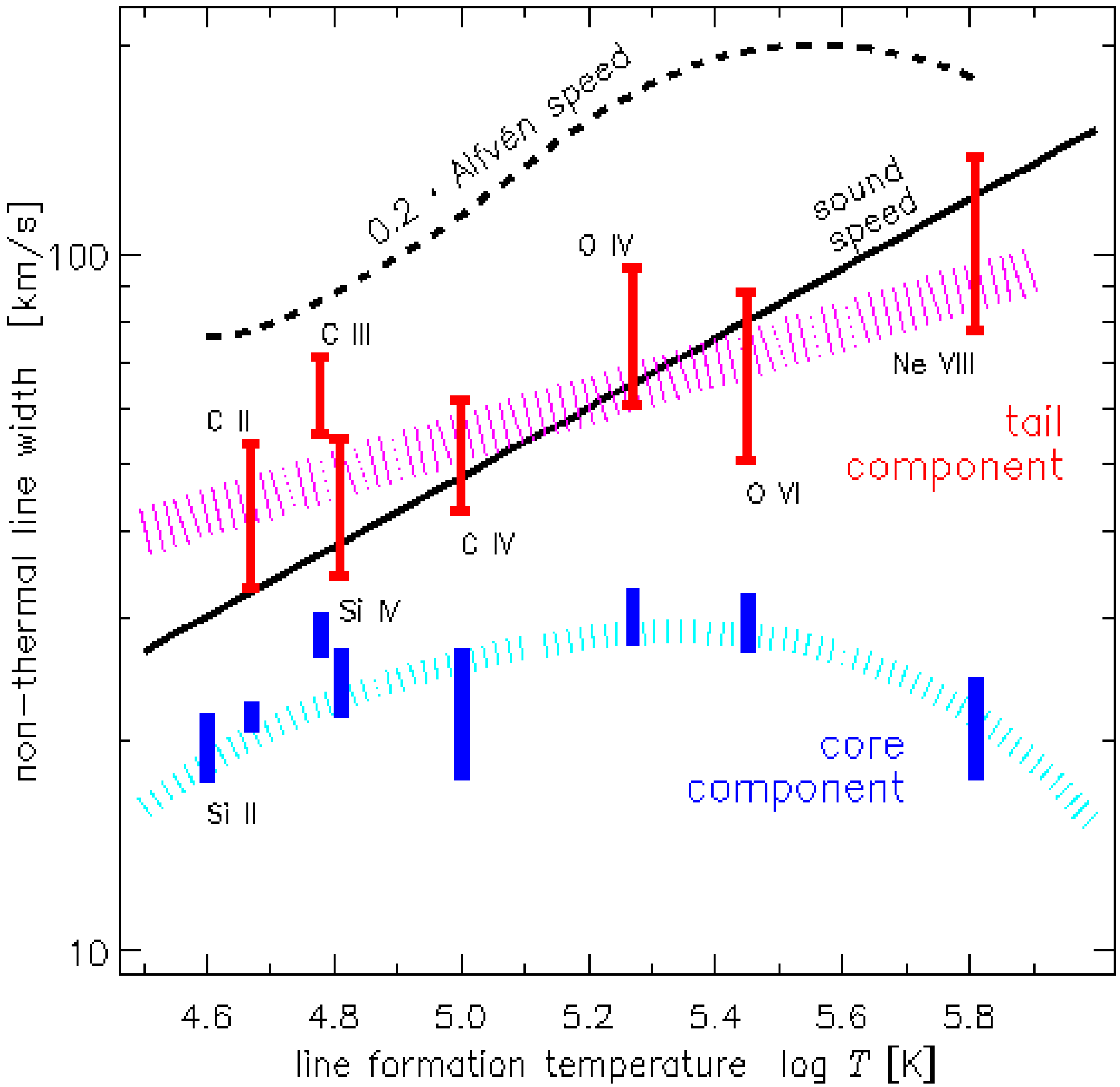
$$T_{i,eff} = T_i + m_i/(2k_B) < \xi^2 >$$

Shortest scales: Proton cyclotron wavelength $\approx$ 100 m in CH

$\lambda_p = 2\pi v_A/\omega_{gp} = 1434$ [km] $(n/cm^{-3})^{-1/2}$, $P_p = 2\pi/\omega_{gp} = 0.66$ [ms] $(B/G)^{-1}$, $\delta V \approx 0.001\ V_A < 1$ km/s.

Largest scales: $\lambda < L$ and $P < T$, where L is the **extent of field of view** and T the **duration of observational sequence**.

Turbulence amplitude versus height



SUMER

Plasma in **funnels**:
Waves escaping on open field lines
--> WKB evolution, with $\xi \sim \rho^{-1/4}$ for Alfvén waves, or weak dissipation......

Plasma in **loops**:
Waves trapped in closed field lines
--> strong dissipation, with loop heating.......

Peter, A&A, **374**, 1108, 2001

Quasi-linear pitch-angle diffusion

Diffusion equation

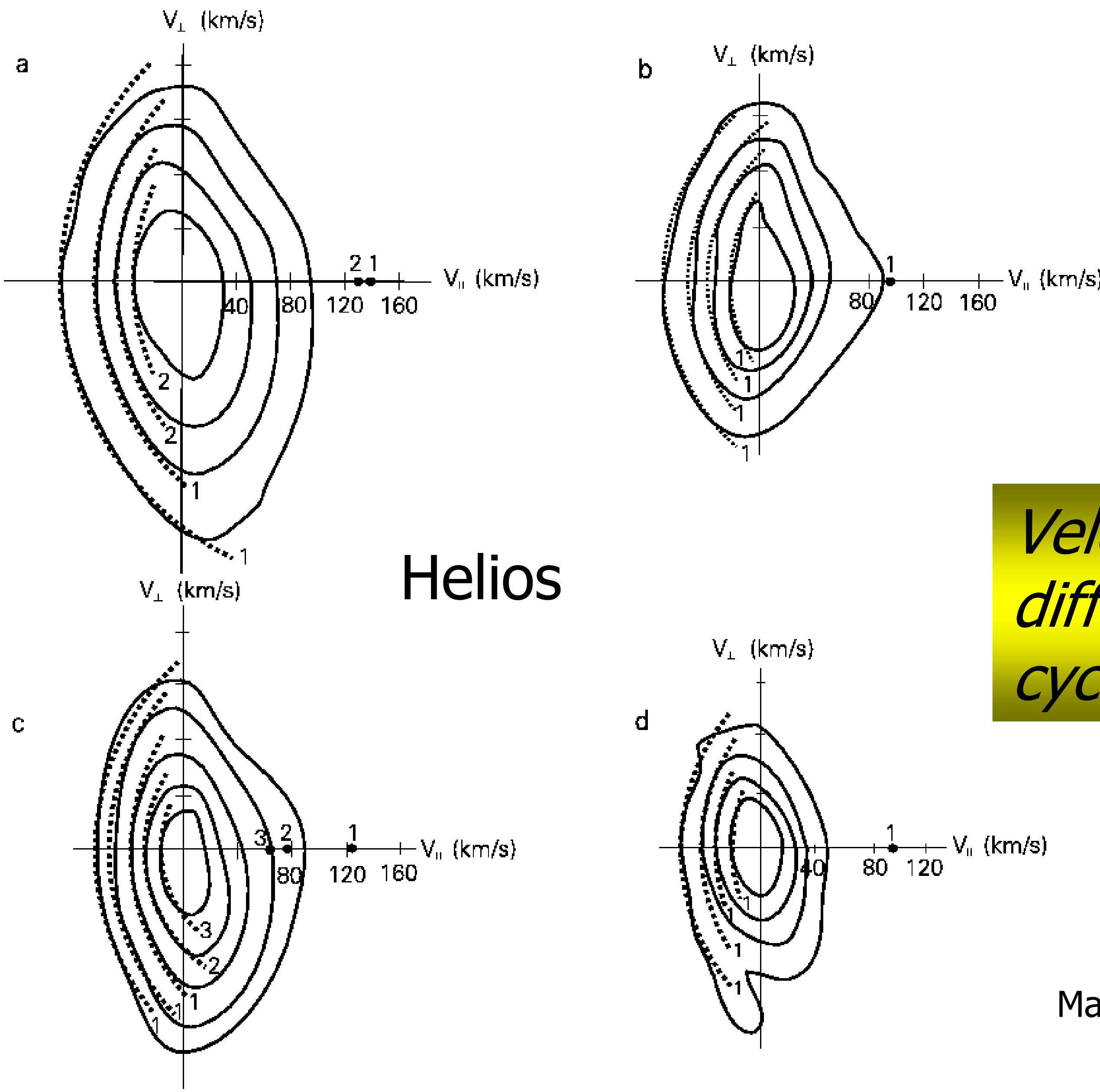
$$\frac{\partial}{\partial t} f_j(V_\perp, V_\parallel, t) = \sum_M \sum_{s=-\infty}^{+\infty} \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \, \mathcal{B}_M(\mathbf{k}) \times \frac{1}{V_\perp} \frac{\partial}{\partial \alpha} \left(V_\perp \, \nu_j(\mathbf{k}, s; V_\parallel, V_\perp) \frac{\partial}{\partial \alpha} f_j(V_\perp, V_\parallel, t) \right)$$

Pitch-angle gradient in wave frame

$$\frac{\partial}{\partial \alpha} = V_\perp \frac{\partial}{\partial V_\parallel} - \left(V_\parallel - \frac{\omega_M(\mathbf{k})}{k_\parallel} \right) \frac{\partial}{\partial V_\perp}$$

Kennel and Engelmann, 1966

Pitch-angle diffusion of protons



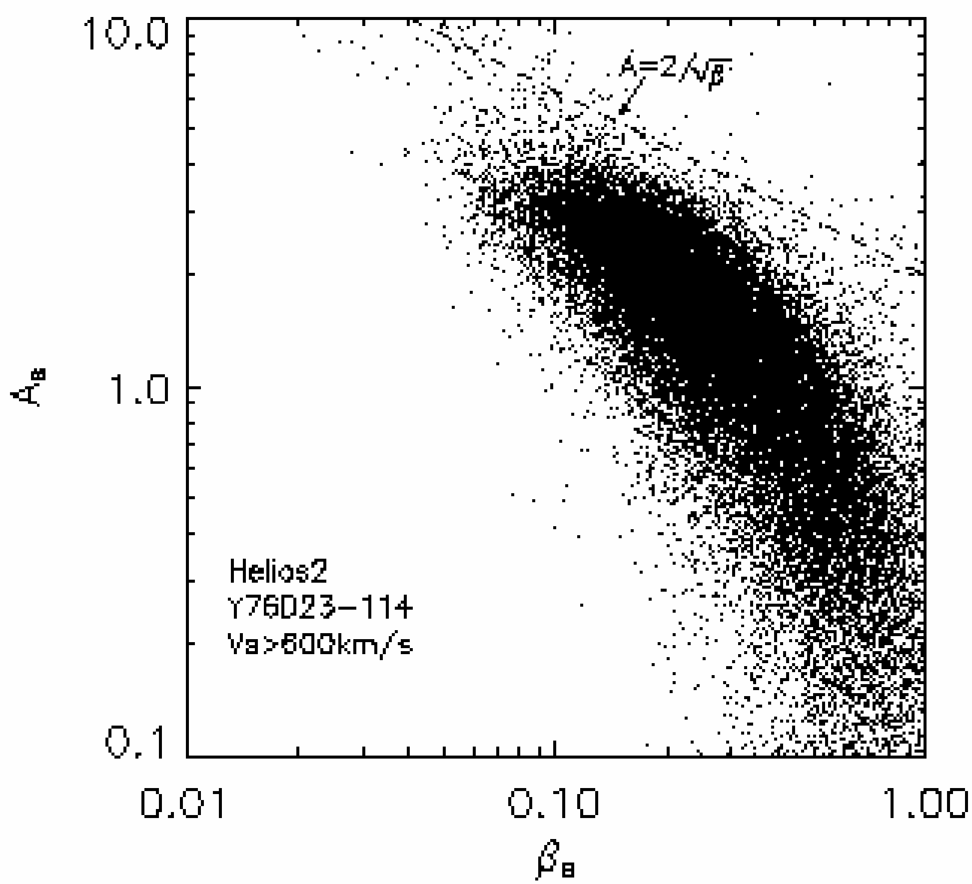
VDF contours are segments of circles centered in the wave frame (< V_A)

Velocity-space resonant diffusion caused by the cyclotron-wave field!

Marsch and Tu, JGR, **106**, 8357, 2001

Anisotropy versus plasma beta

$$A_B = T_{\perp} / T_{||} - 1$$



- Fast solar wind
- V > 600 km/s
- 36297 proton spectra
- Days 23 -114 in 1976

Dashed line refers to plateau:

$$A_B = 2\ \beta_B^{-1/2}$$

$\beta_B = 2\ (W_{||}/V_A)^2\ 1.92$

β_B is the core plasma beta, for VDF values > 20 % of max.

Marsch et al., JGR, **109**, 2004