Numerical solutions for resonantly damped MHD quasi-modes in two-dimensional coronal loops

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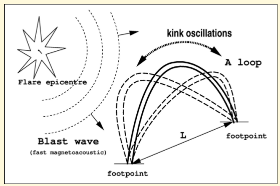
1. Introduction

Context

- Wave-based theories for coronal heating, in particular resonant absorption of waves

Motivation

- No direct observation evidence for resonant absorption
- Direct observational evidence of spatial oscillations in coronal loops triggered by explosive events such as a flare or a filament eruption



Transverse Oscillations of Coronal Loops

Fast kink mode oscillations

$P \simeq 2$–11 mins $\tau_d \simeq 3$–21 mins

- Despite the (relative) simplicity of theoretical models, resonant absorption is able to explain this fast damping and may have an important role in the dissipation of energy in the corona

Purpose

★ Improve theoretical model by considering two-dimensional equilibrium states

$\Longrightarrow$ Important for coronal heating theories based on the dissipation of MHD waves

$\Longrightarrow$ Also important for coronal seismology (determination of unknown coronal physical properties)

2. Equilibrium Configuration

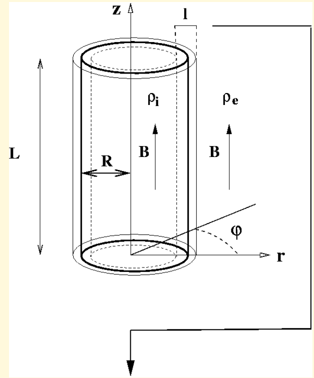
Coronal loop modelled as a gravity-free, straight cylindrically symmetric tube

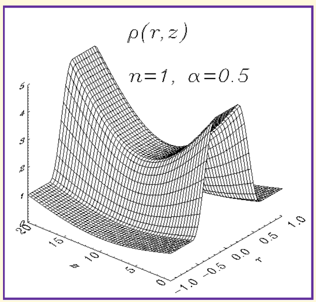
Magnetic field: $\mathbf{B}(r) = B(r)\ \hat{\mathbf{e}}_z$
Cold-plasma approximation $\beta \to 0$

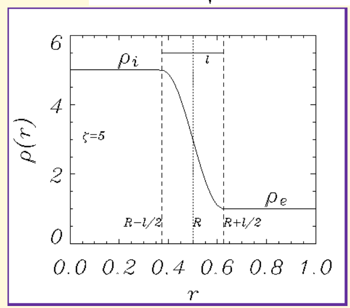
- Magnetic field is constant
- Density $\rho(r,z)$ or Alfvén speed profile $v_A(r,z)$ can be chosen arbitrarily

$$\rho(r,z) = \rho(r)\left[1 - \sum_{n=1}^{+\infty} \alpha_n(r)\sin\left(\frac{n\pi}{L}z\right)\right]$$

$\alpha_n \neq \alpha_n(r)$ and $\alpha_n = 0\ \forall\ n \neq 1$







- Many things missing! For example CURVATURE (see POSTER T. V. DOORSSELAERE)

3. Linear MHD Wave Equations

- The previous equilibrium is then perturbed for the study of small amplitude oscillations around the equilibrium
- Fourier analyse perturbed quantities in t and in the ignorable φ-direction

$$f \sim e^{\imath(m\varphi - \omega t)}$$

- Eigenvalue problem

$$\omega_R = \frac{2\pi}{\text{Period}}$$

$$\omega_I = -\frac{1}{\tau_{\text{damping}}}$$

m=1 Fast kink mode
The only oscillation that displaces the axis of the loop

- Fast kink mode is (irreversibly) damped due to the resonant coupling to torsional Alfvén waves $\Longrightarrow$ quasi-mode
- Our purpose: To compute the frequency and damping rate of these quasi-modes when longitudinal stratification is taken into account

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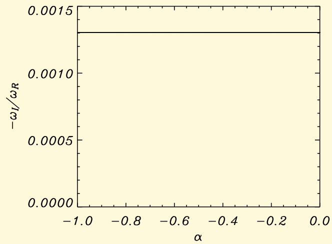
4. Semi-analytical extension of the thin boundary theory to 2D

Generalisation of the thin boundary theory ($l \ll R$) to longitudinally stratified coronal loops Andries et al. (submitted)



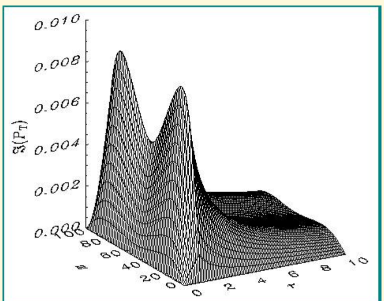
Normalised damping rate $\left(-\frac{\omega_I}{\omega_R} = \frac{1}{2\pi}\frac{P}{\tau_{\text{damping}}}\right)$

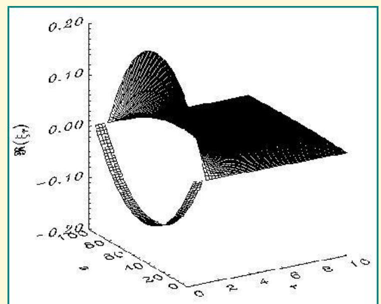
is independent of stratification

when stratification is radially independent

$$\zeta = \frac{\rho_i}{\rho_e} = 2 \quad L = 100R \quad l = 0.01R$$

Higher order harmonic contributions in compression, but not in displacement





(See POSTER J. ANDRIES for details)

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5. Numerical Method

- We use an eigenvalue code POLLUX
- Solves the dissipative linear MHD wave equations
  - Finite elements in the radial direction
  - Spectral method – Longitudinal mode decomposition

$$f\,(r,z) = \sum_{n=-M}^{+M} f_n(r)\; e^{\imath \pi n z/L} \qquad \text{n integer}$$

- Parametric numerical study of the quasi-mode frequency as a function of several parameters

$\Longrightarrow$ $l/R \in [0.0 - 2.0]$: Thickness of the inhomogeneous layer (R=0.02)

$\Longrightarrow$ $\epsilon = \frac{2\pi R}{L} \in [0.02 - 0.18]$: Inverse aspect ratio $\Longleftrightarrow$ Length of the tube

$\Longrightarrow$ $\zeta = \rho_i/\rho_e \in [1.5 - 5.0]$: Density contrast

$\Longrightarrow$ $\alpha \in [0.0 - 1.0]$: Longitudinal stratification coefficient

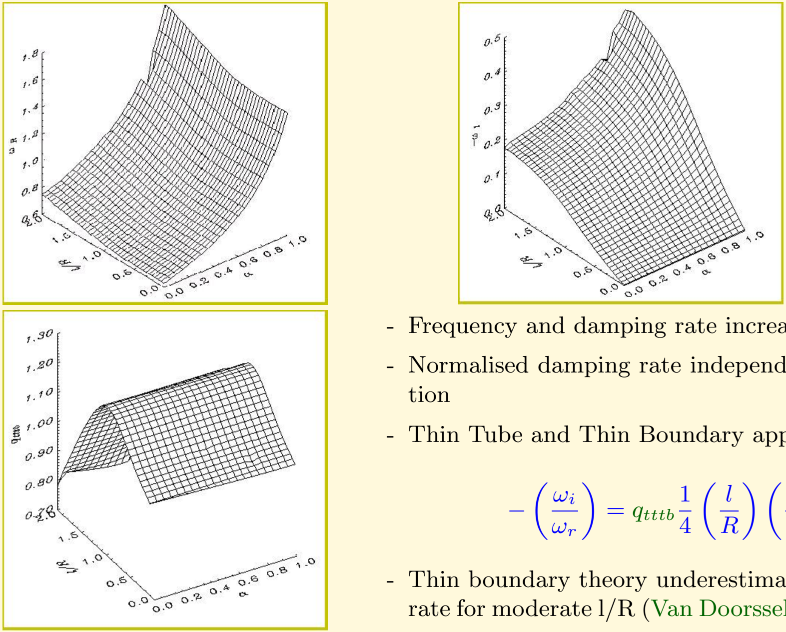
$\Longrightarrow$ η: Resistivity

- – Small enough to assure damping rate is independent of η
- – large enough that number of grid-points suffices to resolve resonant layer.

6. Numerical Results

Thickness of non-uniform layer and Stratification

Consider fixed $\epsilon = 0.02$, $\zeta = 4$



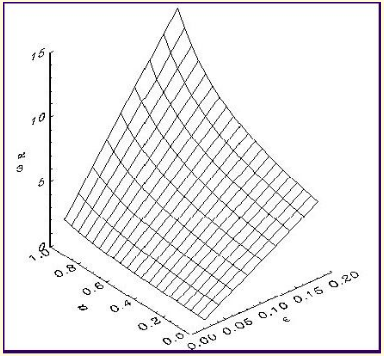
- Frequency and damping rate increase with α
- Normalised damping rate independent of stratification
- Thin Tube and Thin Boundary approximation

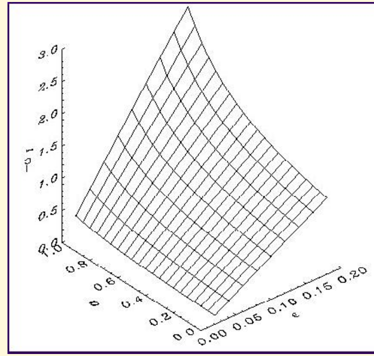
$$-\left(\frac{\omega_i}{\omega_r}\right) = q_{tttb}\frac{1}{4}\left(\frac{l}{R}\right)\left(\frac{\zeta-1}{\zeta+1}\right)$$

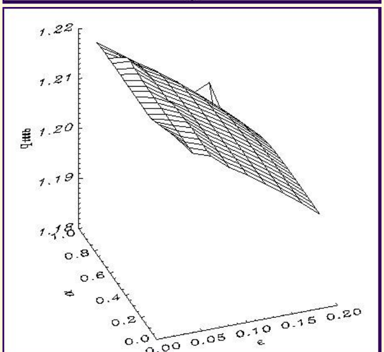
- Thin boundary theory underestimates the damping rate for moderate l/R (Van Doorsselaere et al. 2004)

Length of the loop and Stratification

Consider fixed $l/R = 1$, $\zeta = 5$



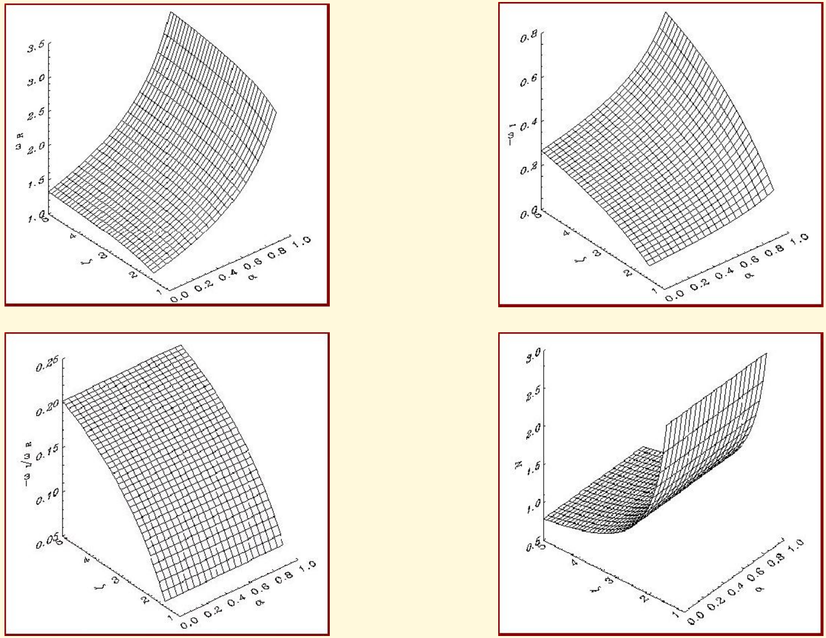




- Weaker damping for shorter loops
- Note that values of $q_{tttb} > 1$ due to the fact that $l/R = 1$

Density contrast and Stratification

Consider fixed $l/R = 1$, $\epsilon = 0.04$



- Density contrast is a very sensitive parameter. The larger ζ the greater the damping
- Large contrast loops get damped in less than a period ($N \in [0.78 - 2.85]$ periods)

7. Summary and Conclusions

Summary

- Numerical solutions for resonantly damped fast kink-mode oscillations have been computed in **two-dimensional** cylindrical models of magnetic coronal loops
- Parametric study of the quasi-mode frequency in a wide range of several parameters (l/R, α, L, ζ)

Conclusions

- Previous results obtained by means of a generalisation of the thin boundary approximations to longitudinally stratified loops are confirmed by numerically solving the dissipative MHD wave equations
- Not limited to thin non-uniform layers
- Longitudinal stratification produces an increase of the period and damping of fast kink-mode oscillations of coronal loop.
- However, the damping per period remains unchanged.
- Signature of coupling between different longitudinal modes shows up in the pressure perturbation total, but not in the displacement.