Current Sheets in the Sun's Corona

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Abstract

Current sheets play an important role in the Sun's atmosphere, especially in coronal heating events and solar flares. They may form in response to motions of the magnetic footpoints in the solar surface or following a loss of equilibrium.

In two dimensions, X-type null points may collapse to current sheets, as described by nonlinear self-similar solutions and by complex-variable theory. Magnetic diffusion resolves the sheets and allows fast reconnection to take place. There are many ways in which such reconnection can occur, depending on the boundary conditions, including one family of *almost-uniform* regimes and another family of *non-uniform* regimes.

In three dimensions, nulls may also collapse to give a growing current along the *spine* or the *fan* of the null. (These are, respectively, isolated field lines or surfaces of field lines that approach or recede from the null point.) Dissipation can then occur by either *spine reconnection* or *fan reconnection*, or also by *separator reconnection* when the current concentrates along, respectively, the spine, the fan or a separator (which is a field line that links one null point to another).

Coronal heating may be produced by reconnection in the following ways: by converging photospheric motions at X-ray bright points; by binary reconnection when pairs of magnetic sources interact; by separator reconnection in complex fields due to tertiary flux interactions; by braiding of field lines; and by coronal tectonics heating at separatrix surfaces between intense flux tubes. Solar flares may occur when a magnetic catastrophe causes the slow eruption of a flux tube, which in turn drives the formation of a current sheet under the flux tube. As the sheet dissipates and reconnects, the overlying field lines holding down the flux tube are released so that rapid eruption and energy release can take place.

The magnetic field of the Sun
Engenders a whole lot of fun;
There's nothing to beat
Collapse to a sheet,
And that's how the heating's begun.

1. Introduction

The Sun's atmosphere has three layers. The surface, called the photosphere, is only 6000K in temperature and it possesses sunspots, dark areas of very strong magnetic field. Images of the line-of-site component of the magnetic field reveal bipolar flux at sunspot pairs and also the presence of concentrated magnetic flux over the whole solar surface at the boundaries of convection cells (supergranules). Above the photosphere lies the warmer and rarer chromosphere and above that the temperature rises dramatically to over a million degrees in the corona. A key problem in astrophysics is then to understand how the corona is heated to such high temperatures.

The energy that is required to heat the corona is about 5000 Wm^{-2} in an active region (a region lying above a group of sunspots where the magnetic field is stronger than normal). The energy flux through the surface of the Sun is plentiful. Since $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, the Poynting flux is of order $\mathbf{E} \times \mathbf{B}/\mu \sim vB^2/\mu$, which is typically 10^4 W m^{-2} for observed velocities (v) of 0.1 km s^{-1} and field strengths (B) of 100G. But the key question is: how is this energy flux converted to heat?

The induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

describes how the magnetic field changes in time. In the corona the ratio (R_m) of the first to the second term on the right is of order 10^8 and so the diffusion term is completely negligible and the magnetic field is frozen to the plasma. The exception is in singularities where the electric current ($\mathbf{j} = \nabla \times \mathbf{B}/\mu$) and therefore the gradient of the magnetic field is very large. So, how does the Sun create such singularities?

In the following sections we describe: the formation and dissipation of 2D current sheets (Sections 2 and 3); the corresponding formation and dissipation in three dimensions (Section 4); and their importance for coronal heating (Section 5) and solar flares (Section 6). For more details of these subjects and for an extensive set of references, see Priest & Forbes 2000.

2. Formation of Current Sheets in Two Dimensions

2.1 X-Points Tend to Collapse (Locally)

If the field lines are free or there is an inflow of energy, an X-point tends to collapse (Figure 2). Physically, if you perturb a flux function from $A = y^2 - x^2$ to $A = y^2 - a^2x^2$ then the $\mathbf{j} \times \mathbf{B}$ force tends to continue the collapse. There are linear solutions with magnetic diffusion, which exhibit a decaying oscillation. Also, nonlinear self-similar solutions to the ideal equations have been studied of the form

$$B_x = a(t)x - b(t)y, \quad B_y = c(t)x + d(t)y,$$

$$v_x = e(t)x + f(t)y, \quad v_y = g(t)x + h(t)y,$$
$$p = i(t)x^2 + j(t)xy + k(t)y^2,$$

which produce a collapse in a finite time, but these are only local.

2.2 Sheet Formation Modelled with a Complex Variable

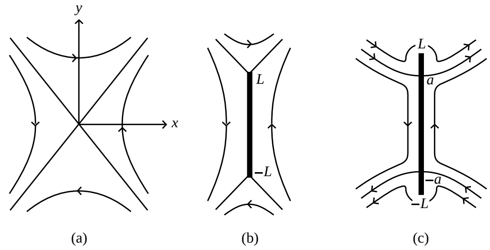

Figure 1. The collapse of an X-point to form a current sheet.

The formation of a sheet has been modelled using complex-variable theory ($z = x + iy$). For example, consider a potential field X-point (Figure 1a) having a field $B_y + iB_x = z$. Move the footpoints of a finite region or add a finite energy to this minimum-energy configuration. Then, if the topology is preserved, to what configuration does this field evolve? Green suggested an evolution to a field of the form (Figure 1b) $B_y + iB_x = (z^2 + L^2)^{1/2}$ having a current sheet stretching from $z = -iL$ to $z = iL$. Later, however, Somov & Syrovatsky 1976 suggested an alternative evolution to $B_y + iB_x = (z^2 + a^2)/(z^2 + L^2)^{1/2}$, which contains singularities at the ends of the sheet (Figure 1c).

2.3 Self-Consistent Formation

More recently, self-consistent solutions satisfying the ideal MHD equations were discovered. Bajer 1990 gave numerical examples using the method of Moffatt 1985 to solve the ideal induction equation (which preserves the topology) and an equation of motion with an artificial damping term that ensures that the motions decay away in time.

Furthermore, an analytical example of time-dependent sheet formation (Priest *et al* 1995) was obtained by expanding the MHD equations in powers of the Alfvén Mach number (M_A), $\mathbf{B} = \mathbf{B}_0 + M_A\mathbf{B}_1 + \ldots$, $\mathbf{v} = M_A\mathbf{v}_0 + \ldots$, so that, to lowest order, the field evolves through a series of quasi-potential states with $\mathbf{j}_0 = \mathbf{0}$ outside current sheets, in such a way that the velocity ($\mathbf{v}_0$) satisfies the ideal induction equation

$\partial \mathbf{B}_0/\partial t = \nabla \times (\mathbf{v_0} \times \mathbf{B_0})$ together with the condition that the acceleration be perpendicular to the field $d\mathbf{v}_0/dt \cdot \mathbf{B}_0 = 0$. One example has singular end-points to the sheet.

2.4 Extensions to the Basic Theory

The basic theory has been extended in many ways. First of all, the form of $B_y + iB_x = f(z)$ has been written down for other configurations. Also, a general method has been proposed for finding $f(z)$ when the topology is preserved (Titov 1992), and techniques have been developed for modelling three-dimensional sheets. The formation of sheets in force-free and magnetostatic fields have been treated (Bajer 1990; Bungey & Priest 1995). Finally, it has been realised that shearing motions of photospheric footpoints can form current sheets along separatrices.

3. Dissipation of Current Sheets in Two Dimensions

When an X-point collapses to a sheet, dissipation allows the field lines to break and reconnect (Figure 1). The magnetic energy is converted to heat, kinetic energy and fast particle energy. This process is the source of heating and of many dynamic processes on the Sun, so how does it occur in both 2D and 3D? In 2D reconnection takes place at an X-type null point, where a localised breakdown of the frozen-field condition produces a change of field-line connectivity. The theory in two dimensions is well developed and includes: slow Sweet-Parker reconnection, fast Petschek reconnection and many other fast regimes that depend on the boundary conditions, such as Almost-Uniform and Nonuniform reconnection.

3.1 Classical Reconnection

Sweet and Parker suggested a model consisting of a simple current sheet with a uniform inflow (Figure 2a). The sheet has length 2L and width 2l, while the inflow velocity and field are v_i and B_i, and the outflow values are v_0 and B_0. Mass conservation into and out of the sheet implies $Lv_i = lv_0$, while a steady balance between inwards advection of flux and its diffusion implies $v_i = \eta/l$. Furthermore, acceleration along the sheet by the $\mathbf{j} \times \mathbf{B}$ force accelerates the plasma to an outflow speed $v_0 = v_A$ equal to the Alfvén speed ($v_A = B_0/(\mu\rho)^{1/2}$). Eliminating L and v_0 between these three relations gives an expression for the reconnection rate (v_i), which may be written in dimensionless form as $M_i = v_i/v_A = 1/R_{mi}^{1/2}$, where the magnetic Reynolds number is $R_{mi} = Lv_A/\eta$. With $R_m = 10^8$, say, the rate is only 10^{-4}, too slow to explain a solar flare.

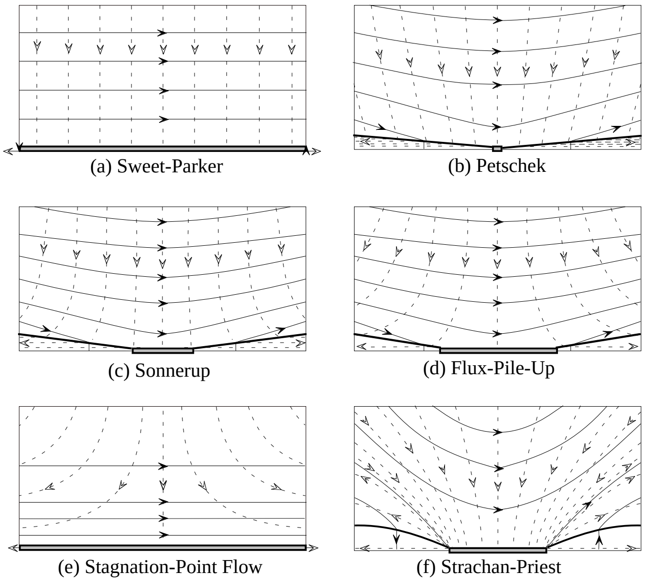

Figure 2. Two-dimensional reconnection regimes: (a) Sweet-Parker, (b) Petschek, (c)-(e) Almost-Uniform and (f) Nonuniform.

Later, Petschek proposed a much more rapid mechanism, in which the sheet bifurcates into two pairs of slow-mode shocks at which most of the energy conversion takes place (Figure 2b). The reconnection speed may be at any rate up to a maximum of $M_e = v_e/v_A = \pi/(8 \log R_{me})$, which is typically 0.01 - 0.1. Here the subscript e refers to "external" values in the inflow region far from the current sheet.

More recently, a family of regimes of Almost-Uniform reconnection was discovered in which the magnetic field lines are weakly curved in the inflow region (Figure 4b-e). The different regimes are produced by different boundary conditions on the inflow, producing either converging or diverging flows. Petschek's mechanism is just one special case.

These models were then generalised even further by allowing the inflowing field lines to be strongly curved (Figure 2f). The resulting Non-Uniform Reconnection regimes again depend on the boundary conditions and agree with numerical experiments provided the same boundary conditions are adopted (and the magnetic diffusivity is nonuniform).

4. Three-Dimensional Reconnection

4.1 Null-Points - Structure and Collapse

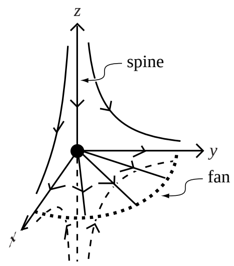

Figure 3. The structure of a null point in three dimensions.

In three dimensions the simplest null point has a field $\mathbf{B} = (x, y, -2z)$, which possesses two families of field lines that pass through the null (Figure 3). An isolated "spine" field line approaches (or recedes from the null), here along the z-axis, while a "fan" surface of field lines emanates from it (or approaches it), here in the xy-plane (Priest & Titov 1996). Most generally, in the linear vicinity of a null the field can be written as

$$\begin{aligned} B_x &= x + \tfrac{1}{2}(q - j_{||})y \\ B_y &= \tfrac{1}{2}(q + j_{||})x + py \\ B_z &= j_{\perp}y - (p+1)z \end{aligned}$$

in terms of four parameters, namely (p, q) and the components $(j_{||}, j_{\perp})$ of the current along the spine and normal to it (Parnell *et al* 1996).

As in two dimensions, nulls can collapse to give a growing current along the spine or in the fan (Bulanov & Sakai 1997; Parnell *et al* 1996). There are linear and nonlinear treatments, which show the development of a singularity in a finite time, but the analysis is only local. When the fan current grows, the angle between the fan and spine decreases, whereas when the spine current grows the field lines in the fan rotate and close up (Figure 4).

4.2 Reconnection at a 3D Null

At 3D null points there are three types of reconnection (Figures 5 and 6), namely spine reconnection (when the current focuses along the spine), fan reconnection (when it is concentrated in the fan) and separator reconnection, when the current grows along the separator joining one null to another (Priest & Titov 1996; Longcope 1996; Hornig &

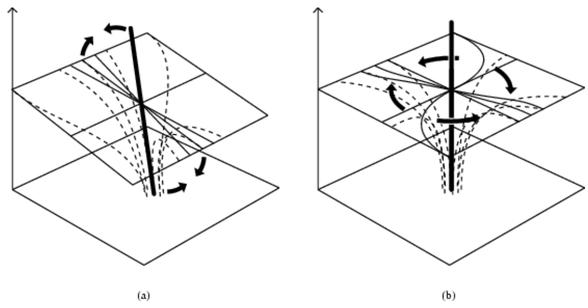

Figure 4. An example of (a) a fan current growing and (b) a spine current growing.

Schindler 1996). Figure 5 shows the result of a kinematic analysis, while Figure 6 gives an image from a 3D resistive MHD experiment of Galsgaard & Nordlund 1997a in which they explored the effect of imposing a shear on two boundaries of a box initially containing eight null points in equilibrium.

5. Can Reconnection Heat the Solar Corona?

The solar corona is highly complex. It consists of three types of structure, which may well be heated differently. Coronal holes are magnetically open into interplanetary space, so the fast solar wind escapes from them, whereas coronal loops have both ends attached to the solar surface. There are also tiny points of emission called X-ray bright points, several hundred of which are present at any time.

Reconnection may be heating in different ways. Simple reconnection may be driven at null points by photospheric motions to create an X-ray bright point according to the Converging Flux model. In this model new flux emerges in a supergranulation cell and migrates to the cell boundary, where it reconnects with pre-existing flux on the boundary.

A second way is by "binary reconnection" (Priest & Schrijver 1999) when pairs of oppositely directed flux sources in the solar surface move relative to one another and drive reconnection in the corona. Interactions between three or more sources in complex fields may instead involve separator reconnection (Priest & Titov 1996; Longcope 1996). Finally, braiding of field lines may also produce current sheets and heating.

The relative motion of two photospheric sources in an overlying field has been studied numerically by Galsgaard *et al* 2000. Initially, the sources are unconnected, but as they move past one another reconnection takes place at a separator (Figure 7) and creates a twisted flux tube joining the two sources. Braiding was first proposed by Parker 1979 as

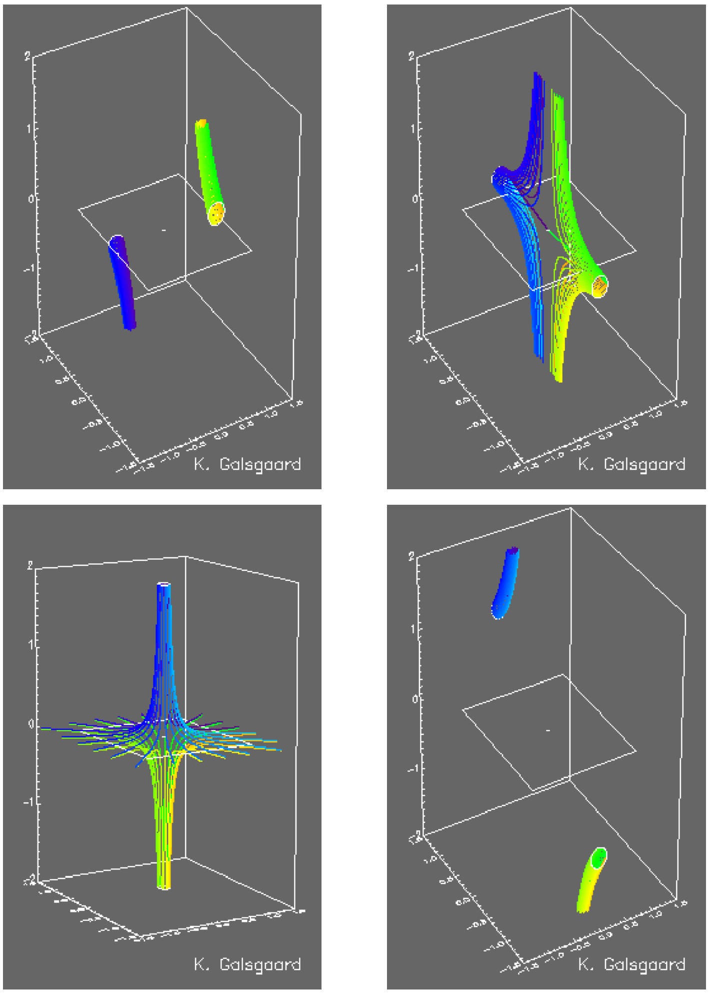

Figure 5. Two flux tubes (a) approaching one another and then interacting by (b) spine and (c) fan reconnection, and finally (d) receding from each other.

a heating model and has been extensively studied (see Nordlund and Bhattacharjee). In particular, an experiment by Galsgaard & Nordlund 1997b shows the growth of current sheets and an impulsive heating.

6. How Do Solar Flares and Coronal Mass Ejections Occur?

The overall picture for a solar flare is that during the pre-flare phase a large flux tube with an overlying arcade of field lines becomes sheared

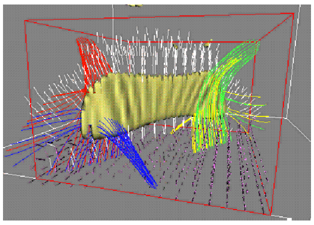

Figure 6. The magnetic field lines, fluid velocity vectors (white) and electric current (central grey surface) in separator reconnection (Galsgaard & Nordlund 1997a).

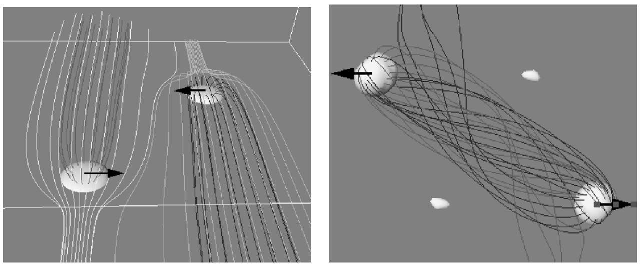

Figure 7. Numerical experiment showing (a) the initial magnetic field lines coming from two sources and (b) the interaction during separator reconnection (Galsgaard *et al* 2000).

and rises slowly. At some critical point, reconnection of the overlying arcade occurs at points below the rising flux tube. This disconnects some of the overlying field lines from the solar surface and so allows a rapid eruption to take place. As the eruption and reconnection continue, an arcade of rising hot loops is created with bright ribbons at its feet.

A model in two dimensions for the cause of the eruption has been developed (Priest & Forbes 2000), consisting of a flux tube with an overlying arcade having sources that are a distance 2λ apart. As λ decreases and the sources slowly approach one another, the height of the flux tube slowly decreases and the configuration passes through a series of equilibria - until, that is, a critical point is reached beyond which there is no neighbouring equilibrium. If there is no reconnection, the flux tube rises towards a new equilibrium that contains a current sheet linking the tube to the solar surface. However, in numerical experiments, reconnection is driven at the current sheet and allows a rapid eruption of the tube. More recently, a three-dimensional version of this model has been developed.

7. Conclusion

On the Sun it is likely that the occurrence of current sheets is responsible both for coronal heating and for solar flares. In two dimensions, the formation and dissipation of current sheets is well understood, but in three dimensions the theory is only just beginning to be developed. However, there are many fascinating observations and numerical experiments to guide and inspire us in the future.

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