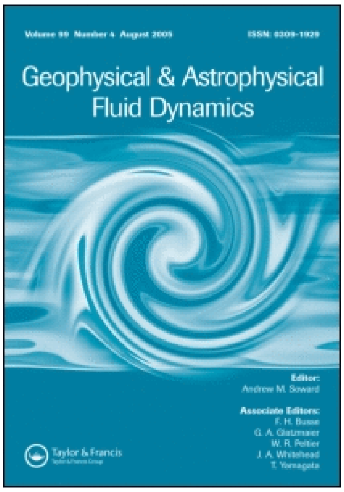

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THE ROLES OF ADVECTION AND DIFFUSION IN PLANAR MAGNETIC MERGING SOLUTIONS

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Since cosmic plasmas are highly conducting, large-scale magnetic fields are tied almost completely to the velocity field of the fluid. Only in localized regions of strong current density can the magnetic field slip through the plasma, allowing magnetic energy to be converted into ohmic heating or the kinetic energy of mass motion. Here we contrast the roles of advection and resistive diffusion in three different steady-state two-dimensional models for magnetic energy conversion: magnetic annihilation, reconnective diffusion and a kinematic model based on the classical magnetic reconnection picture. First we examine the diagnostic of 'field-line slippage' and show that it provides a useful indicator of the relative importance of advection and diffusion in each solution. We then quantify the energy release characteristics of the different models by examining the ratio of ohmic heat to kinetic energy generation.

Keywords: Magnetic reconnection; plasmas; MHD

1. INTRODUCTION

The conversion of magnetic energy into heat and kinetic energy plays an important role in many astrophysical processes. In particular, in the Sun's corona the magnetic field is believed to be the energy source for solar flares, prominences, coronal heating and a host of other

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interesting phenomena. Although the importance of the magnetic field as an energy source has been generally accepted for over 50 years, the exact mechanisms by which the energy is liberated are still not well understood.

It is generally accepted that magnetic merging provides the simplest model for magnetic energy release. Even so, it is only in the last few years that exact steady-state incompressible solutions have been developed for planar reconnection (Craig and Henton, 1995). Previous models were restricted to magnetic annihilation, that is, the merging of *straight field-lines* (*e.g.*, Sonnerup and Priest, 1975). In magnetic annihilation energy is dissipated directly by ohmic heating – there is no transfer of kinetic energy to the fluid. The new generation of reconnection solutions make it possible to explore more general release mechanisms. This requires diagnostics to indicate whether magnetic energy is liberated primarily as ohmic heating *in situ*, or whether it is transferred to the fluid as the kinetic energy of mass motion.

We begin by considering the magnetic annihilation solution of Sonnerup and Priest (1975), in which two anti-parallel regions of magnetic field are swept together by the plasma flow and destroy one another. We then discuss the reconnection solution of Craig and Henton (1995). We label this model 'reconnective diffusion' since it exhibits advection across one separatrix (like the classical reconnection models), but diffusion across the other separatrix (like the diffusive annihilation solutions). It has the interesting feature of breaking the usual reconnection symmetries by invoking shear flows across a global current layer. By contrast, the classical reconnection picture has advection across both field separatrices and involves the cutting and rejoining of field-lines within a localized diffusion region surrounding the neutral point. It something of an irony that there is no convenient description of the classical reconnection picture. Faced with this difficulty we adopt, for the purposes of a making a qualitative comparison, an approximate kinematic solution.

We discuss the various energy release models using the diagnostic of field-line slippage, which measures the discrepancy between the flow of the plasma and the motion of individual field-lines. Regions where there are large amounts of slippage signal a breakdown of the frozen-in condition, where the field is tied to the plasma. These are important as potential sites for magnetic energy release *via* reconnection and annihilation. We also show that field-line slippage can be related to the Poynting flux of energy through the plasma. This in turn reflects the conversion of magnetic energy into heat and the kinetic energy of mass motion.

2. MHD EQUATIONS

The equations that govern the behavior of a steady-state, incompressible magnetized plasma can be writtein the following dimensionless form

$$(\mathbf{v}\cdot\nabla)\boldsymbol{\omega}-(\boldsymbol{\omega}\cdot\nabla)\mathbf{v}=(\mathbf{B}\cdot\nabla)\mathbf{J}-(\mathbf{J}\cdot\nabla)\mathbf{B}, \tag{2.1}$$

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{J}, \tag{2.2}$$

$$\nabla\cdot\mathbf{B}=0, \tag{2.3}$$

$$\nabla\cdot\mathbf{v}=0, \tag{2.4}$$

where $\mathbf{v}$ is the plasma velocity, $\mathbf{B}$ is the magnetic field, $\boldsymbol{\omega}=\nabla\times\mathbf{v}$ is the vorticity, $\mathbf{J}=\nabla\times\mathbf{B}$ is the current density, $\mathbf{E}$ is the electric field and η is the dimensionless resistivity. The resistivity for astrophysical plasmas is typically very small: its magnitude is the inverse Lundquist number, of order 10^{-12} for the solar corona. As we limit our attention to two-dimensional solutions – restricted to the $x-y$ plane, say – we note that the electric field $\mathbf{E}$ must be directed in the z-direction. Since $\mathbf{E}$ can be written as the gradient of a scalar function, this implies $\mathbf{E}$ must be a constant vector whose magnitude is independent of position. The magnitude of $\mathbf{E}$ controls the rate of energy conversion.

To calculate the slippage velocity of the field-lines for a given solution, we first determine the component of the plasma velocity perpendicular to the magnetic field by crossing (2.2) with $\mathbf{B}$, to give

$$\mathbf{v}_\perp=\frac{(\mathbf{E}-\eta\mathbf{J})}{B^2}\times\mathbf{B}. \tag{2.5}$$

This perpendicular component is the only component of the plasma velocity that is relevant to determining the slippage, as fluid motions along the field-lines play no part in decoupling the field from the flow and releasing energy. The field-line velocities themselves are then determined from the expression

$$\mathbf{v}_f = \frac{\mathbf{E}\times\mathbf{B}}{B^2}. \tag{2.6}$$

This field-line velocity is just the velocity that the plasma would have if resistive effects were absent, and the plasma could be treated as ideal. Note that this velocity becomes infinite in regions where the magnetic field vanishes or the electric field is singular – the case of a singular electric field can only arise in three dimensions where $\mathbf{E}$ is allowed a spatial dependence, see Priest and Titov (1996). The slippage velocity is defined as the difference between these two velocity measures, namely

$$\mathbf{v}_s = \mathbf{v}_f - \mathbf{v}_\perp = \frac{\eta\mathbf{J}\times\mathbf{B}}{B^2}. \tag{2.7}$$

The slippage also becomes infinite at neutral points of the magnetic field, since $\mathbf{J}$ will typically be nonzero at such points. This reflects the fact that at a neutral point the field and the flow can completely decouple, and magnetic field can be destroyed.

3. THE MODELS

3.1. Magnetic Annihilation

Magnetic annihilation solutions based on stagnation-point flow have been constructed by Clarke (1964) and Sonnerup and Priest (1975). More recently, there has been an upsurge of interest in annihilation based on the inclusion of global shear flows into the velocity field (Besser *et al.*, 1990; Phan and Sonnerup, 1990; Anderson and Priest 1993; Jardine *et al.*, 1993; Jardine, 1994). Since we are interested mainly in the generic features of the merging process we restrict attention to the original planar model of Sonnerup and Priest (1975).

The solution is found by assuming velocity and magnetic field profiles of the form

$$\mathbf{v} = (-\alpha x, \alpha y, 0), \quad \mathbf{B} = [0, B(x), 0]. \tag{3.1}$$

On substituting these forms into (2.1)–(2.4) we find a solution in which the magnetic field vanishes at the stagnation-point, namely

$$B(x) = \frac{E}{\eta\mu}\operatorname{daw}(\mu x), \tag{3.2}$$

where E is the magnitude of the electric field $\mathbf{E} = E\hat{\mathbf{z}}$, $\mu^2 = \alpha/(2\eta)$ and daw(x) is the Dawson function (Spanier and Oldham, 1987), given by

$$\operatorname{daw}(x) = \int_0^x \exp(t^2 - x^2)\,dt.$$

This solution represents the mutual annihilation of two equal and opposite anti-parallel fields, see Figure 1(a). In the limit of small resistivity, η, the solution possesses a strong current sheet centered over the flow stagnation-point. The sheet has a thickness in the x-direction proportional to $\eta^{1/2}$, but extends to infinity in the y-direction.

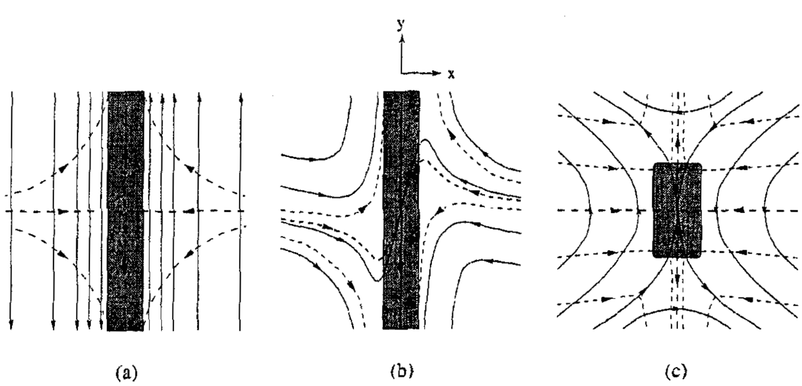

FIGURE 1 Models for (a) magnetic annihilation, (b) reconnective diffusion and (c) classical magnetic reconnection, showing the magnetic field-lines (solid), streamlines (dashed) and high current region (shaded).

3.2. Reconnective Diffusion

The Craig and Henton (1995) analysis considers velocity and magnetic fields of the form

$$\mathbf{v} = \mathbf{P} + \mathbf{f}(x), \quad \mathbf{B} = \lambda\mathbf{P} + \mathbf{g}(x), \tag{3.3}$$

where λ is a scalar superposition constant and $\mathbf{P}(x, y)$ is an arbitrary potential field. They show that only the stagation-point form $\mathbf{P} = (-\alpha x, \alpha y, 0)$ allows an exact solution in two-dimensions: to within a linear shear component, the disturbance fields are related by $\mathbf{f} = \lambda\mathbf{g}$. It is possible, however, as Watson and Craig (1997a, b) emphasize, to construct approximate, quasi-steady solutions by allowing $\mathbf{P}(x, y)$ to represent non-potential global fields. These methods also extend to the analytical construction of time-dependent, three-dimensional solutions (*e.g.*, Craig and McClymont, 1997; Craig *et al.*, 1997).

Here we consider only the basic solution

$$\begin{aligned} \mathbf{v} &= [-\alpha x, \alpha y + \lambda g(x), 0], \\ \mathbf{B} &= [-\lambda\alpha x, \lambda\alpha y + g(x), 0], \\ g(x) &= \frac{E}{\eta\bar{\mu}}\mathrm{daw}(\bar{\mu}x), \end{aligned} \tag{3.4}$$

where $\bar{\mu}^2 = (1 - \lambda^2)\alpha/(2\eta)$. A schematic diagram of a solution of this form is shown in Figure 1(b). Like the Sonnerup and Priest annihilation model – recovered by setting $\lambda = 0$ – this solution possesses an infinite current sheet aligned with the y-axis. But in general the field-lines are curved, allowing reconnection at the neutral point. This can be confirmed in Figure 1(b) by noting that plasma can flow across the curved field separatrix.

3.3. Classical Reconnection

The classical picture of reconnection, summarized in Figure 1(c), has been widely accepted by many years. Both the flow and the magnetic field are symmetric about the x- and y-axes, and the diffusion region – the region of strong currents – is localized to a finite volume surrounding the flow stagnation-point. Because of the complexity of this two-dimensional, nonlinear resistive MHD process, it is not surprising that no exact analytical solution is available to model it, at least under the present assumptions of steady-state, incompressible merging. However, circumstantial evidence to support the model is available from many sources: for instance, from time-dependent, compressible X-point theory (Craig and McClymont, 1991; Hassman, 1992; McClymont and Craig, 1996), from quasi-steady and time-dependent numerical simulations (*e.g.*, Biskamp, 1994), and from semi-analytical, quasi-linear analysis (Priest and Forbes, 1986, 1992).

In what follows we present an approximate kinematic model that mimics the basic features of the traditional reconnection picture. Although the kinematic treatment is incapable of representing certain phenomena adequately – for example, the likelihood of extended currents along the magnetic separatrices – we note that the properties of the flow profile deduced below should hold for any localized current solution. This is independent, of course, of the plausibility of the localized current model.

We begin by choosing a magnetic field profile that possesses a suitably localized current distribution. We then use (2.2), along with boundary conditions appropriate to the imposed symmetries, to calculate the plasma flow that supports the magnetic field.

Our choice for the magnetic field, written in terms of a magnetic flux function $\psi(x,y)$, where $\mathbf{B} = \nabla \times (\psi\hat{\mathbf{z}})$, is of the form

$$\psi(x,y) = \{k^2(x^2 - y^2) + \ln[f(x,y)]\}/200, \tag{3.5}$$

where $f(x,y)$ is a polynomial function of x and y chosen to ensure that the current density is adequately localized, and the normalization factor is included so that J at the origin is comparable to the value in the annihilation and reconnective diffusion models. By taking

$$f(x,y) = 1 + k^2x^2 + k^2y^2 + \frac{1}{5}k^4x^4 + \frac{14}{5}k^4x^2y^2 + \frac{1}{5}k^4y^4, \tag{3.6}$$

we obtain a current sheet of thickness $1/k$. If we choose k so that $E = \eta J$ is satisfied at the origin then (with E constant) the sheet thickness scales as $\eta^{1/2}$. This form also ensures that the field osculates at the origin, as required for the standard reconnection symmetries, see Priest and Cowley (1975) and Biskamp (1994).

With the magnetic field specified, the velocity stream function can be determined by integrating (numerically) along the characteristics. The symmetry and incompressibility conditions on the flow allow a unique solution for $\mathbf{v}$ to be constructed. Writing $\mathbf{v} = \nabla\times(\phi\hat{\mathbf{z}})$ allows (2.2) to be written in the form

$$E + \mathbf{B}\cdot\nabla\phi = \eta J, \tag{3.7}$$

which implies that on a given field-line

$$\frac{d\phi}{ds} = \frac{\eta J - E}{B}, \tag{3.8}$$

where s is distance along the field-line. We can now solve for the stream function ϕ in the first quadrant ($x \geq 0, y \geq 0$) by starting at points that lie on the lines $x = 0$ and $y = 0$ (which correspond to $\phi = 0$, due to the symmetry conditions) and then stepping out along the field-lines associated with these points, while at the same time integrating (3.8) for ϕ. In practice we implement this procedure by solving (3.8) and equations for the field-line, parameterized in terms of s,

$$\frac{dx}{ds} = -B_x/B, \quad \frac{dy}{ds} = -B_y/B, \tag{3.9}$$

as a system for the three variables x, y and ϕ. [The choice of signs in (3.9) merely defines the direction in which s increases along the field lines].

A solution generated using this procedure is displayed in Figure 2. Panel (a) shows the magnetic field, (b) the current density and (c) the stream-lines. Note the sharp turn-around in the flow, centered about the magnetic field separatrix.

As Figure 2 implies, we have constructed a classical reconnection model only at the expense of having a strong vortex layer overlying the magnetic separatrix. This is common to many reconnection solutions (Soward and Priest, 1986; Priest et al., 1994). Recent work (Titov and Priest, 1997) suggests that viscous effects may allow the vortex-current separatrix layer to be confined rather than to stretch out to infinity.

More critical is the absence of resistivity. We observe from (3.8) that the stream function is unbounded at the origin (where $B = 0$) and that this singularity is propagated out along the field separatrix. Physically,

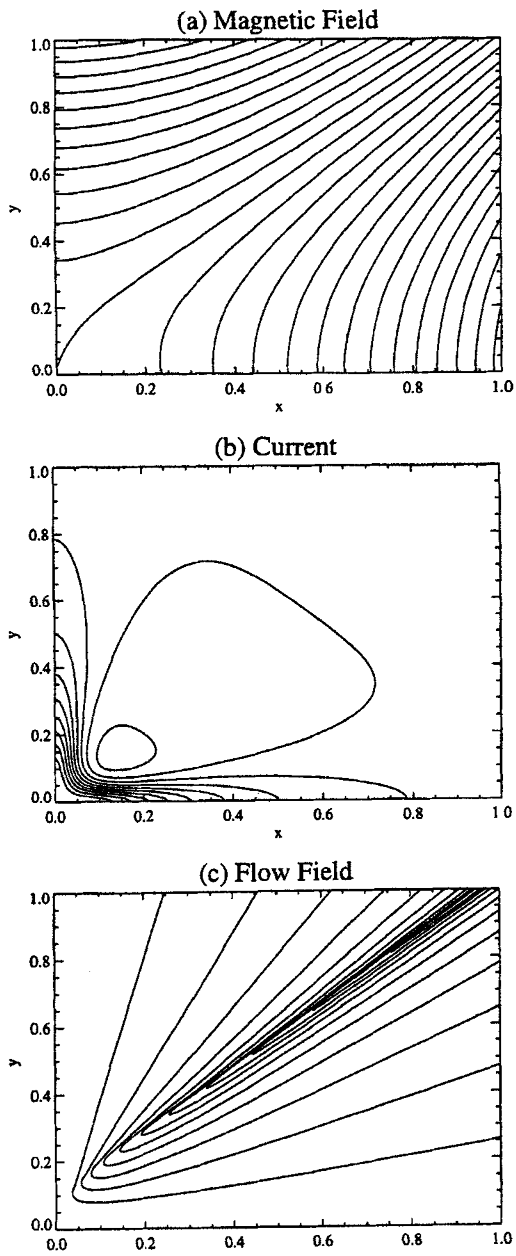

FIGURE 2 A solution of the kinematic classical reconnection model. Having chosen a magnetic field of the form (3.4), with $k = 10$, $\eta = 0.05$, $E = -0.1$, we then solve (3.6) for the streamfunction ϕ. Note that the sharp turnaround in the flow is aligned with the separatrix of the magnetic field.

we know that diffusion must be present at the origin to allow reconnection to occur. The characteristic formulation shows that only those field-lines that thread the current region can be effective in resolving the discontinuity in the flow. Thus, in a highly localized current region, only those field-lines sufficiently close to the separatrix can contribute to smoothing the singularity. It follows that the vortex layer overlying the separatrix must become stronger and stronger with the increasing localization of the current region.

4. COMPARISON OF THE MODELS

4.1. The Roles of Advection and Diffusion

The most obvious difference between the three models is the breaking of one of the symmetries in the reconnective diffusion model. This violation of the standard symmetries allows both the field and the flow separatrices to remain hyperbolic in the vicinity of the neutral point, unlike the classical picture of reconnection, which requires the field separatrices to osculate.

Other differences arise due to the differing roles of advection and diffusion in each model. The annihilation solution of Figure 1(a) is relatively straightforward to interpret physically, but the reconnection models can be clarified most easily by focusing on 'snapshots' of individual field-lines at successive instants. (Individual field-lines are constantly moving through the picture of course, despite the fact that these are steady-state solutions and the overall picture remains constant in time).

Figure 3 shows the time evolution of field-lines for both reconnective diffusion and classical reconnection. The position of a given field-line is marked at four evenly spaced time intervals (labeled 1–4). In both cases the field-line labeled 3 indicates the time when the field-line passes through the neutral point and reconnection occurs. In Figure 3(a) we see that both advection and diffusion play important roles in transporting field-lines across the separatrix structures (diffusion must be responsible for transporting flux across the straight vertical separatrix, as there is no plasma flow across this line) and hence we term the underlying process reconnective diffusion. Figure 3(b), however,

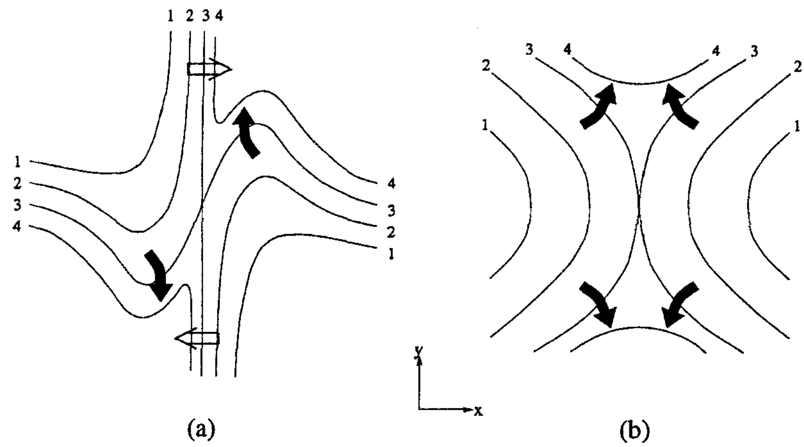

FIGURE 3 A diagram showing the time evolution of a given field-line at four evenly spaced time intervals, labelled 1–4, for (a) reconnective diffusion and (b) classical magnetic reconnection. In (a) magnetic field-lines are advected across the curved field separatrix (solid arrows), but must diffuse (hollow arrows) across the straight separatrix, as there is no plasma flow across this surface. The field-lines in (b) on the other hand are predominantly advected across all the field separatrices.

shows that advection is the dominant process by which field-lines cross the separatrices outside the central diffusion region in the classical reconnection picture.

4.2. Field-line Slippage

We can go some way to confirming these interpretations by comparing the slippage velocities for the different models. Fortunately, it is a simple task to construct exact expressions for the slippage in all three models, since all we require is the functional form of the magnetic field, see Eq. (2.7). The results are displayed in Figure 4–slippage velocities in the vicinity of neutral points have been excluded from the diagrams, since they become infinite.

If we examine the simple annihilation model first [Fig. 4(a)], we find that there are two distinct regions on either side of the neutral line: an outer advection-dominated region, where the plasma moves slightly faster than the field, and the net slippage is outward; and an inner diffusion region, where the plasma flow stalls, but the field-lines continue to accelerate until they collide and annihilate at the neutral

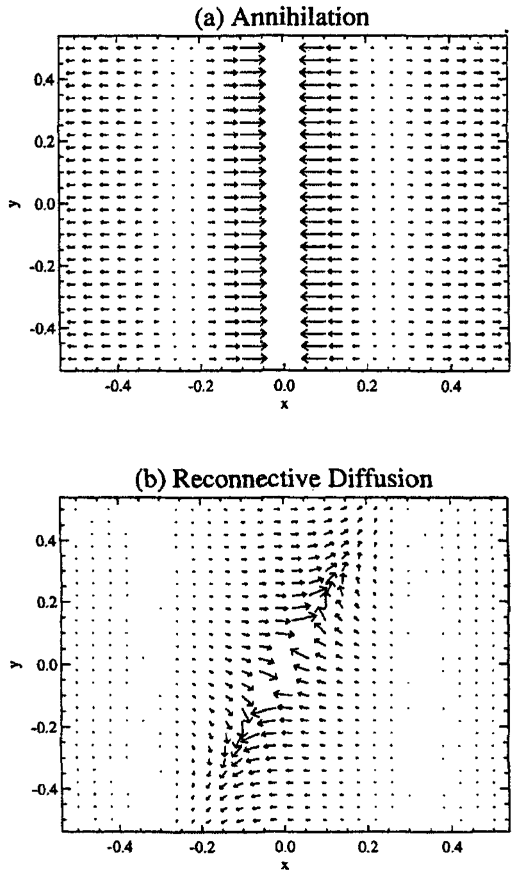

FIGURE 4 The slippage velocity associated with each of three basic models: (a) An annihilation solutions with $E = -0.1$, $\eta = 0.05$ and $\alpha = 1.5$, (b) a reconnective diffusion solution $E = -0.1$, $\eta = 0.05$, $\alpha = 1.5$ and $\lambda = 2/3$ and (c) a classical reconnection solution with $E = -0.1$, $\eta = 0.05$ and $k = 10$. Slippage velocities near the neutral points have been excluded, as they become infinite, and each diagram focuses on a small region centered about the neutral point.

line. The weak outward slippage in the outer field eventually tends to zero as one moves out away from the neutral line and the field becomes effectively frozen in. This outward slippage is an artifact of the reverse currents present in the Sonnerup and Priest annihilation model and it would not be present in a model that did not possess this feature, *e.g.*, the numerical results of Biskamp (1986) – reverse currents do

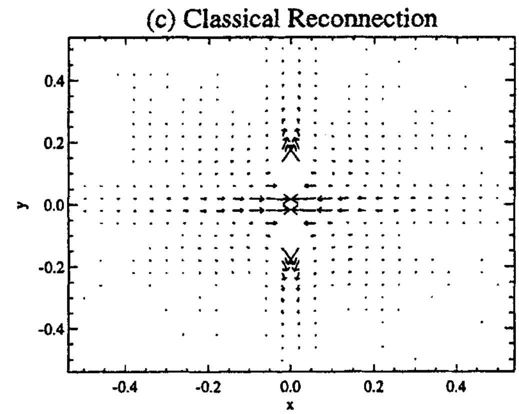

FIGURE 4 (Continued).

appear in Biskamp's solutions, but they occur at the ends of the sheet, rather than across its width.

The first thing to note about the slippage in the reconnective diffusion model [Fig. 4(b)] is that there is indeed slippage across the vertical separatrix of the field, and this confirms that diffusion is acting to transport magnetic field across the y-axis. However, the bulk of the slippage occurring within the high current region results from a magnetic slingshot effect, see Figure 5. This effect operates mid-way between the two field separatrices, and it is the result of magnetic tension acting to straighten out strongly curved field-lines – these field-lines then spring out away from the neutral point dragging plasma with them. Because of the presence of strong currents in this region the field and the fluid are only weakly coupled, and slippage occurs as the plasma lags behind the field.

The diagram for the slippage in the classical reconnection model [Fig. 4(c)] contains few surprises. The only regions of appreciable slippage are localized about the current spike at the neutral point. As with the annihilation model the main slippage occurs as plasma decelerates into the stagnation-point. However, there is now also slippage associated with a weak magnetic slingshot, where exhaust plasma is slung out of the reconnection region along the y-axis. Another feature of this picture is that there is no slippage associated with the magnetic separatrices – implying that advection is the dominant

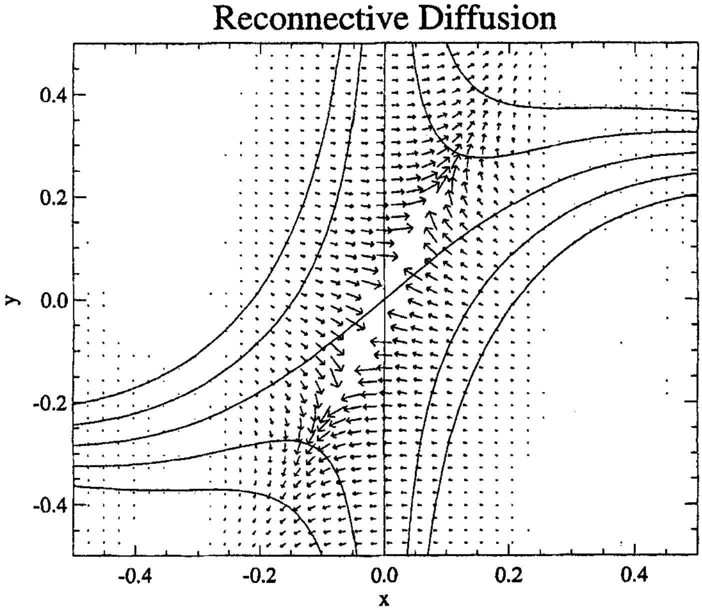

FIGURE 5 A blow-up of Figure 4(b) showing the slippage associated with the reconnective diffusion model. Overlaying the slippage vectors are some magnetic field-lines that surround the neutral point. Note that the bulk of the slippage occurs in a region lying between the two field separatrices. This region acts as a magnetic slingshot, where highly curved field-lines fling material out away from the neutral point.

process that transports magnetic flux across these layers. The lack of slippage across the separatrices may simply be an artifact of our kinematic model. In a true dynamic model of classical reconnection, where the momentum Eq. (2.1) must also be satisfied, separatrix currents would probably result in order to counter the sharp turnaround in the flow that occurs along the magnetic separatrices. These separatrix currents could give rise to appreciable slippage – the slippage scales as $v_s \approx \eta J/B$ – at least provided J remains significant along the separatrices.

4.3. Energetic Considerations

As already mentioned, we are interested in whether the magnetic energy release is manifested in strong plasma heating or in the kinetic energy of mass motion. In the case of magnetic annihilation the situation is unambiguous: no fluid flows are induced by the magnetic collapse so we need only evaluate the ohmic heating term $W_\eta = \langle \eta J^2 \rangle$, where the angle brackets refer to integration over the volume. It is easily shown that, for fixed inflow conditions on $x = \pm 1$, the energy release is fast in the sense that W_η scales independently of any positive power of η: specifically, $W_\eta \sim \eta^{-1/2}$. This does not imply, however, that unbounded dissipation rates can be maintained in the limit of small η–for the hydromagnetic pressure in the sheet must be limited to physically plausible values (see Inverarity and Priest, 1996; Watson and Craig, 1997a; Craig *et al.*, 1997).

Although the reconnective diffusion model retains the ohmic heating properties of the simple annihilation model (as it is constructed from it), there is now a magnetic slingshot that transfers energy between the field and the flow. To quantify the energy transfer it is instructive to consider the time dependent forms

$$\begin{aligned} \mathbf{v} &= [-\alpha x, \alpha y + v(x,t), 0], \\ \mathbf{B} &= [-\lambda \alpha x, \lambda \alpha y + b(x,t), 0], \end{aligned} \tag{4.1}$$

before specializing to the steady state solution $v(x, t) \rightarrow \lambda g(x)$ and $b(x, t) \rightarrow g(x)$. By dotting the momentum equation with $\mathbf{v}$ and the induction equation with $\mathbf{B}$ we find

$$\frac{d}{dt}\left\langle \frac{1}{2} v^2 \right\rangle = 2\alpha v_1^2 - 3\alpha \left\langle \frac{1}{2} v^2 \right\rangle + \lambda\alpha\langle vb \rangle - \lambda\alpha\langle xvb' \rangle, \tag{4.2}$$

$$\begin{aligned} \frac{d}{dt}\left\langle \frac{1}{2} b^2 \right\rangle = {} & 2\alpha b_1^2 + \alpha \left\langle \frac{1}{2} b^2 \right\rangle - \lambda\alpha\langle vb \rangle - \lambda\alpha\langle xv'b \rangle \\ & + 4\eta b_1 b_1' - \eta\langle J^2 \rangle, \end{aligned} \tag{4.3}$$

where the angle brackets refer to integration over $(-1, 1) \times (-1, 1)$ and v_1, b_1 and b_1' are the values of v, b and b' on the boundary $x = 1$ (these results assume that antisymmetric solutions are taken for v and b).

The annihilation model is recovered from this system by setting $\lambda = v = 0$. In this case there are no contributions to the kinetic energy equation, and the magnetic energy equation reduces to

$$\frac{d}{dt}\left\langle \frac{1}{2} b^2 \right\rangle = 2\alpha b_1^2 + \alpha \left\langle \frac{1}{2} b^2 \right\rangle + 4\eta b_1 b_1' - \eta\langle J^2 \rangle. \tag{4.4}$$

In a steady state the boundary terms are $O(\eta)$ and can be neglected when η is small. Therefore $\alpha\langle b^2/2\rangle$, which represents the increase in magnetic energy due to the stretching and bunching of the field by the flow, must balance $\eta\langle J^2\rangle$, which represents the energy loss due to ohmic heating of the plasma, see also Clarke (1964).

If we allow v and λ to be nonzero we then have a fully reconnective solution. The stretching and ohmic heating terms remain, but additional energy terms arise. The most important new terms from the present viewpoint are the $\lambda\alpha\langle vb\rangle$ terms that appear with opposite signs in each of (4.2) and (4.3). These terms quantify the energy interchange between the field and the flow. In the steady state the interchange term reduces to $I = \lambda^2\alpha\langle g^2\rangle$. For well behaved reconnection solutions we require $\alpha > 0$ and $|\lambda| < 1$. In this case I represents a growth term for the kinetic energy and a decay term for the magnetic energy, which corresponds to the magnetic sling shot that accelerates plasma away from the neutral point. The remaining terms in (4.2) and (4.3) refer to work done by the fluid against the pressure gradient and energy leakage out of the top and bottom of the box. The leakage terms (the two angle bracket terms containing derivatives) correspond to shear waves exiting through $y = \pm 1$, see Craig and McClymont (1997).

What is the relative importance of ohmic heating and kinetic energy generation in the reconnection model? We have already stated that the ohmic dissipation rate scales as $W_\eta \sim \eta^{-1/2}$, and we find that the energy interchange term also possesses this scaling, *i.e.*, $I \sim \lambda^2\alpha\eta^{-1/2}$. Therefore comparable amounts of magnetic energy go into heating and accelerating the plasma, at least provided $\lambda^2\alpha \sim O(1)$.

Unfortunately we cannot perform comparable energy calculations for the classical reconnection model because the solution is purely kinematic. However, we can make an observation based on the fact that the magnetic field must osculate close to the origin. Since, the tangential field at the ends of the sheet scales as $B_x \sim y^3$, as opposed to $B_x \sim y$ for a classical X-point – see Biskamp (1994) – we must conclude that the magnetic slingshot effect is relatively weak. This suggests that the classical reconnection model may be closer to the pure annihilation model in that it generates strong plasma heating, but only gives rise to relatively weak fluid flows.

We should caution, however, that more complete treatments of the classical reconnection problem show phenomena not represented in

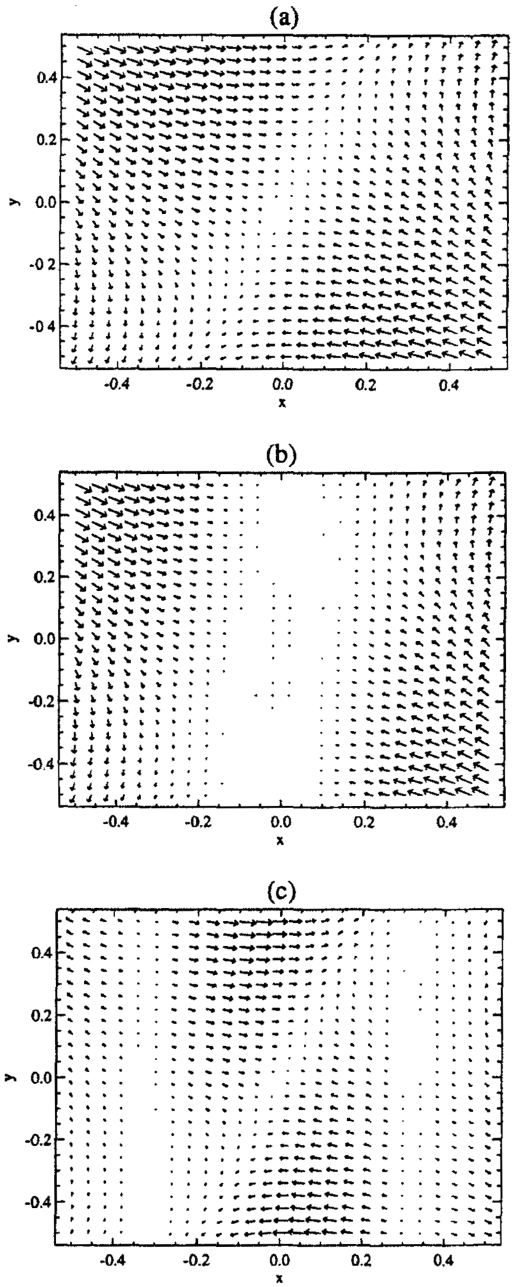

FIGURE 6 A diagram of the Poynting flux for the reconnective diffusion model of Figures 4(b) and 5, (a) gives the total Poynting flux, whilst (b) and (c) represent the contributions from advection and diffusion respectively.

our simple kinematic treatment. Particularly noteworthy in the numerical experiments of Biskamp (1986, 1994) are strong jets exiting the ends of the current sheets. These jets are driven by total (gas plus magnetic) pressure gradients, and not a magnetic slingshot (in fact in these cases the magnetic field acts as a brake on the flow, giving rise to the characteristic weak reverse currents at the ends of the sheet). Unfortunately Biskamp's experiments do not include scalings for the strength of these jets, so it is very much an open question whether classical reconnection scenarios can give rise to significant kinetic energy generation.

4.4. The Poynting Flux

A further diagnostic that we might monitor is the Poynting flux $\mathbf{P} = \mathbf{E}\times\mathbf{B}$, a quantity which measures the flow of electromagnetic energy in each solution. This quantity is obviously very closely related to the field-line velocity (2.6). Rather than examining Poynting fluxes for all three solutions we focus on the Poynting flux for the case of reconnective diffusion. Figure 6(a) shows the total Poynting flux for this case, while Figures 6(b) and (c) show the contributions to this quantity from advection and diffusion respectively. These results reinforce our earlier interpretation of this solution based on of field-line slippage. Not surprisingly we find that the magnetic slingshot acts as a sink for magnetic energy – magnetic energy is transported into the slingshot region both by advection and diffusion and there it is converted into kinetic energy. Furthermore we see from Figure 6(c) that diffusion allows magnetic energy to flow cross the vertical field separatrix. These effects are also highlighted in the field-line slippage diagnostic, which we believe provides a clearer signature of reconnection.

5. CONCLUSIONS

We have examined the roles of advection and diffusion in three different models for magnetic energy release. The annihilation model of Sonnerup and Priest (1975), the reconnective diffusion model of Craig and Henton (1995) and a simple kinematic model of classical reconnection. The annihilation model is open to a particularly simple interpretation. Advection dominates in the outer field, and acts to localize anti-parallel magnetic field about $x = 0$. Within the current sheet that forms, diffusion dominates, allowing flux to cancel across the neutral line. In this model there is no back-reaction of the field on the flow, consequently there is no conversion of magnetic energy into mass motion, and energy is dissipated from the system purely by ohmic heating.

The two reconnection models have a complicated structure and are therefore difficult to interpret. In the classical reconnection picture, advection is the more important effect in the outer field – diffusion only plays a significant role in the vicinity of the stagnation-point. Furthermore, owing to the osculation of the magnetic field in this model, the magnetic slingshots at the ends of the current sheet are weak. In our simple kinematic model this fact implies that there is little interchange between magnetic and kinetic energy. Admittedly, in a more realistic dynamic model, pressure gradients along the sheet might give rise to significant mass flows, but this remains an open question.

Reconnective diffusion is the model with the richest structure. Fortunately, the existence of an exact solution allows us to give a more detailed interpretation. Diffusion again dominates in the immediate vicinity of the neutral point, but it is now also important all the way along the y-axis, where it transports flux across the vertical magnetic field separatrix, which we therefore term the "diffusion separatrix". In contrast, outside the high current region advection is clearly the dominant process by which magnetic field moves across the curved field separatrix, which we call the "advection separatrix".

In order to compare the various models studied here it has proved useful to examine the field-line slippage velocity associated with each case. We have shown that this measure, given by (2.7), provides a useful diagnostic for investigating two-dimensional reconnective solutions. Presumably this diagnostic can be applied unambiguously to three-dimensional reconnection solutions, where the motion of field-lines is much harder to interpret.

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