THEORY OF 3D RECONNECTION AND CORONAL HEATING

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ABSTRACT

Magnetic reconnection is a prime candidate for heating coronae. We here summarise recent developments in the theory of 3D reconnection and the different ways in which it is thought to heat the solar corona. At 3D null points reconnection may occur by either spine, fan or separator reconnection. In absence of null points, reconnection of two flux tubes is completely different from 2D reconnection, since in general in 3D a flux-conserving velocity \( w \) does not exist. Instead two flux tubes split into four parts, which 'flip' via 'virtual flux tubes' to form four flux tubes. Heating in the corona has been proposed to occur by: driven reconnection at X-ray bright points; binary reconnection due to the coronal interaction of a pair of opposite-polarity magnetic sources; separator reconnection due to a higher-order interaction; braiding; and coronal tectonics due to the formation and dissipation of current sheets at myriads of separatix surfaces that thread the corona and separate the flux coming from the many different sources in the photosphere.

INTRODUCTION

The equations of MHD comprise Newton’s equation of motion together with equations of continuity and energy and the induction equation

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B, \tag{1}
\]

which is obtained by taking the curl of Ohm’s Law

\[
E + v \times B = j/\sigma. \tag{2}
\]

In most of the solar atmosphere the second term on the right of (1) is negligible and so (2) reduces to

\[
E + v \times B = 0.
\]

The magnetic field is frozen to the plasma and hangs on to its energy. The exception is in singularities where the electric current \( j = \nabla \times B/\mu \) and magnetic gradient became extraordinarily large. These singularities form at null points and at the interface between two magnetic flux tubes as they reconnect.

In two dimensions X-type null points tend to collapse and reconnect in ways that are now fairly well understood (e.g., Priest and Forbes, 2000). However, in three dimensions we are only just beginning to explore the ways in which reconnection can work. Here we describe some new insights into 3D reconnection (Section 2) and a new suggestion for coronal heating that has just been proposed (Section 3).

RECONNECTION IN THREE DIMENSIONS

In three dimensions the simplest linear null point has a field

\[
B = (x, y, -2z).
\]
such that \( \nabla \cdot \mathbf{B} = 0 \). It possesses two families of field lines through the null point, namely, an isolated *spine* field line which approaches the origin along the z-axis and a *fan* surface of field lines which radiate from the origin in the xy-plane.

Three types of reconnection can occur at a null point (Priest and Titov, 1996), namely: *spine reconnection*, where the current and the dissipation concentrate along the spine; *fan reconnection*, where it focusses on the fan; and *separator reconnection*, where it is localised along the separator field line joining one null point to another.

Let us compare the way reconnection operates in two and three dimensions. In two dimensions, in order of magnitude, the Sweet-Parker relations determine the reconnection rate for a diffusion region of thickness \( 2l \) and width \( 2L \) as

\[
V_i v_i = -\frac{v_A}{\eta}
\]

where \( R_m = L v_A / \eta \) is the global magnetic Reynolds number.

In three dimensions the comparable argument for a disc of radius \( L \) and thickness \( l \) between two flux tubes inclined at an angle \( \theta \) (Figure 1) gives a reconnection rate of

\[
v_i = \frac{v_A}{R_m^{1/2}} \sqrt{2 \sin(\theta/2)},
\]

which varies with \( \theta \) and has a maximum value that is \( \sqrt{2} \) times bigger than the Sweet-Parker value when the field lines are oppositely directed (\( \theta = \pi \)). A second difference between 2D and 3D arises when one considers the mapping of field lines. In 2D near an X-point, if a footpoint \( A_1 \) is moved across a separatrix to a position \( A_2 \), there is a discontinuity in the mapping from a point \( B_1 \) to a point \( D_2 \) (Figure 2). However, in 3D, if a uniform field, say, is added in the z-direction to the field of an X-point, as we move from \( A_1 \), to \( A_2 \) in Figure 3, the mapping is continuous as the point \( B_1 \) flips rapidly across the top of the box from \( B_1 \) to \( D_2 \). At the same time as \( C_1 \) moves to \( C_2 \), the other end of the field line flips across the top of the box from \( D_1 \) to \( B_2 \).

A third, and much more profound difference between 2D and 3D arises when one tries to describe reconnection in terms of the motion of field lines or flux tubes (Hornig, 2002). In 2D it is always possible to calculate the velocity \( (\mathbf{w}) \) of flux tubes from the equation
Fig. 2. The mapping of field lines in two dimensions near an X-type null point. $A_1$ maps to $B_1$ while a nearby point $A_2$ maps to point $D_2$ that is far from $B_1$.

Fig. 3. The mapping of field lines in three dimensions in the absence of a null point. $A_1$ maps to $B_1$, and, as $A_1$ moves to a nearby point $A_2$, $B_1$, moves rapidly but continuously across the top of the box to $D_2$.

Fig. 4. The reconnection of flux tubes in 2D. Outside the diffusion region the tubes move with a velocity ($w$) equal to the plasma velocity ($v$). The effect of reconnection is to make the initial four half-tubes to break and reconnect to form two flux tubes.
and 2 flux tubes move in towards an X-point, break at one point and are rejoined to one another to form 2 tubes that move out at a velocity $w = v$ (Figure 4).

However in 3D, it may be proved (Priest, Hornig and Pontin, 2003) that a flux tube velocity ($w$) satisfying (2) does not in general exist. Field lines continually change their connections in a diffusion region and two tubes reconnect, with each half of the original 2 tubes joining a new tube to form 4 flux tubes (Figure 5). During the process of reconnection, the two original flux tubes split to form 4 parts, each of which flips and is joined not to one tube moving at velocity $v$ but to a virtual tube, whose location moves at $w$: i.e., it is joined to a series of different tubes as time proceeds (Figure 6).

A numerical experiment on the reconnection of initially straight tubes by a stagnation-point flow reveals extra features of 3D reconnection (Linton and Priest, 2003). The reconnection is fragmented and occurs at several points. Also, it creates complex twisted and braided flux tubes (Figure 7). A simple comparison of the initial mutual magnetic helicity ($-F^2$) of the initial straight tubes of flux $F$ and the final self-helicity ($\Phi F^2/(2\pi)$), of each of two final tubes of twist $\Phi$ leads to a value of $\Phi = -\pi$ for the final twist.

**CAN RECONNECTION HEAT THE CORONA?**

Yes, probably in different ways. X-ray bright points are likely to be heated after flux emerges and reconnects at the boundaries of supergranules (Parnell et al, 1994). Coronal loops may possibly be heated
Fig. 7. The twisted and braided flux tubes formed by reconnection during a 3D resistive MHD numerical experiment from two initially straight tubes inclined at $\pi/2$ (Linton and Priest, 2003)

Fig. 8. A schematic of a coronal loop consisting of many sub volumes, each linked to a separate source and divided from one another by separatrix surfaces.

by: binary reconnection (Priest et al, 2003), when two unbalanced magnetic sources interact; separator reconnection (Longcope, 1996, 1998), when several sources interact at a separator, or by braiding (Parker, 1979).

However, there is an alternative that I would like to describe here. What is the effect of the magnetic carpet on coronal heating? Three factors are important: the coronal magnetic field comes through the solar surface in concentrated sources; these flux sources are highly dynamic, so that in the quiet Sun the flux is reprocessed every 14 hours (Hagenaar, 2001); the global topology of a complex field consists of a collection of topologically separate volumes divided from one another by separatrix surfaces.

A Coronal Tectonics Model for coronal heating (Priest, Heyvaerts and Title, 2002) takes account of these three factors about the magnetic carpet. Each coronal loop has a magnetic field that links the solar surface in many sources. The flux from each source is topologically distinct and is separated from each other by separatrix surfaces (Figure 8). As the sources move, the coronal magnetic field slips and forms current sheets along the separatrices, which then reconnect and heat. Thus, in our view, the corona is filled with myriads of separatrix current sheets.

But the fundamental flux units in the photosphere are likely to be intense flux tubes with fields of 1200 G,
diameters of 100 km (or less) and fluxes of $3 \times 10^{17}$ Mx (or less). A single X-ray bright point thus links to a hundred sources and each TRACE loop probably consists of at least 10 finer, as yet unresolved, loops.

Whereas Parker's braiding model assumes complex footpoint motions acting on a uniform field, we consider the effect of simple motions on an array of flux tubes that are anchored in small discrete sources. For a simple model consisting of an array of flux tubes anchored in two parallel planes (Figure 9), we have demonstrated the formation of current sheets and estimated the heating. A more realistic model would have the sources asymmetrically placed so as to create many more separatrices, or more realistic still, it would place all the sources on one plane and have mixed polarity. The basic principles would, however, be unchanged.

The results give a uniform heating along each separatrix, so each (sub-telescopic) coronal flux tube would be heated uniformly. But 90% of the photospheric flux closes low down in the magnetic carpet (Close et al, 2003), so the remaining 10% forms large-scale connections. Thus, the magnetic carpet would be heated more effectively than the large-scale corona. Unresolved observations of coronal loops would give enhanced heating near the loop feet in the carpet, while the upper parts of coronal loops would be heated uniformly but less strongly.

CONCLUSION

Three-dimensional reconnection is very different from reconnection in two dimensions. We propose here that flux tubes may reconnect by splitting and then each part flipping in a virtual flow during which it is connected to a series of flux tubes. Coronal Heating may occur by several reconnection mechanisms, namely, binary reconnection, separator reconnection, braiding or coronal tectonics, in which topologically separate fields slip at myriads of separatrix surfaces and drive reconnection heating.

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