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# The Topology of Coronal Magnetic Fields in Active Regions

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**Abstract.** In order to understand processes on the Sun which are dominated by the magnetic field, it is important to understand the structure of the magnetic field itself. The sources of the coronal field are magnetic fragments scattered over the solar surface and clustered around the edges of supergranule cells. These sources are not static but continually moving, coalescing, fragmenting and cancelling with one another. The resulting magnetic field has an incredibly complex topology. In order to begin to understand this complexity we consider the magnetic field arising from a small number of discrete sources and study the topological behaviour exhibited by this system.

## 1. Introduction

The *topological skeleton* of a complex magnetic field $\boldsymbol{B}(\boldsymbol{r})$ is defined by the configuration of the sources, the null points and a network of spine field lines and separatrix fan surfaces (Priest, Bungey and Titov, 1997). The null points of the system satisfy the equation

$$\boldsymbol{B}(\boldsymbol{r}) = \mathbf{o}, \tag{1}$$

and in the generic case the linearised field near the null possesses three distinct eigenvalues which sum to zero in view of the equation $\nabla . \boldsymbol{B} = 0$. The field lines are given by the equation

$$\frac{d\boldsymbol{r}}{ds} = \frac{\boldsymbol{B}(\boldsymbol{r})}{|\boldsymbol{B}(\boldsymbol{r})|} \tag{2}$$

or

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} = \frac{ds}{|\boldsymbol{B}|}. \tag{3}$$

The nature of the different kinds of null has been classified by Priest & Titov (1996) and Lau & Finn (1990). When two eigenvalues are positive, the plane of the corresponding eigenvectors consists of a separatrix surface of field lines radiating out from the null. The eigenvector of the third (negative) eigenvalue is an isolated field line, known as the spine field line, which approaches the null from two opposite directions. When two eigenvalues are negative and one positive, the fan field lines in the separatrix surface converge into the null and the spine field line is directed away from the null. The separatrix surfaces divide the space into several topologically distinct regions in which the field lines have the same sources and sinks.

The states may be topologically stable or unstable. A state is topologically stable if any small perturbation in the parameters does not change the state. Topologically unstable states occur when variations of a parameter cause the system to bifurcate between two stable states. The bifurcations may be either local, when null points either coalesce or fragment, or global when the null points do not appear or disappear.

## 2. Formulation of the Model

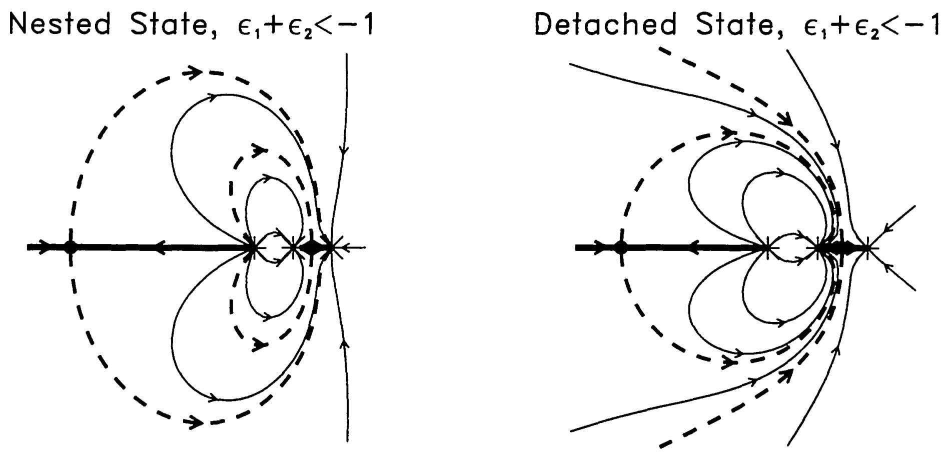


Figure 1. Examples of topological states, showing the field lines which lie in the $z = 0$ plane. The stars are sources, the dots are nulls, the dashed lines are separatrices and the bold lines are spine field lines.

### 2.1. Equations

Consider the potential coronal magnetic field produced by a series of flux sources on the photosphere, which is treated as a plane. The coronal magnetic field depends on the positions and strengths of the sources. Without loss of generality, the first source can be placed at the origin and given a source strength of 1. The next source can be placed a distance 1 along the x-axis and the position of the third is scaled relative to the first two. The second and third source strengths are scaled relative to that of the first source. As each source obeys an inverse square law, the rescaled magnetic field is given by

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} + \epsilon_1 \frac{\boldsymbol{r}-\boldsymbol{i}}{|\boldsymbol{r}-\boldsymbol{i}|^3} + \epsilon_2 \frac{\boldsymbol{r}-a\boldsymbol{r}_2}{|\boldsymbol{r}-a\boldsymbol{r}_2|^3}, \tag{4}$$

where $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $\boldsymbol{r}_2 = \cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}$.

### 2.2. Topological States

It transpires that the three-source system has eight stable topological states associated with it, which we refer to as the separate state, the touching state, the enclosed state, the nested state, the intersecting state, the detached state, the divided state and the triangular state. Two of these refer to the case where all sources have the same polarity, while the other six refer to the case where one source (say the source at the origin) has polarity opposite to the other two. Examples of some of these topological states are shown in Figure 1 and Figure 3a,c,d and f. The skeletons include separatrices (dashed) and spines (dark solid) in the $z = 0$ plane. Some general field lines are also plotted in order to indicate the nature of the fields in each region.

## 3. Local Bifurcation of the System

When the angular separation of sources ($\theta$) varies, the system can bifurcate locally or globally. As an example, consider local bifurcations between the separate state, the touching state and the enclosed state when $-0.5 < \epsilon_1, \epsilon_2 < 0$ and $a \approx 1$ (note that $a = 1$ is a special, non-generic case, however).

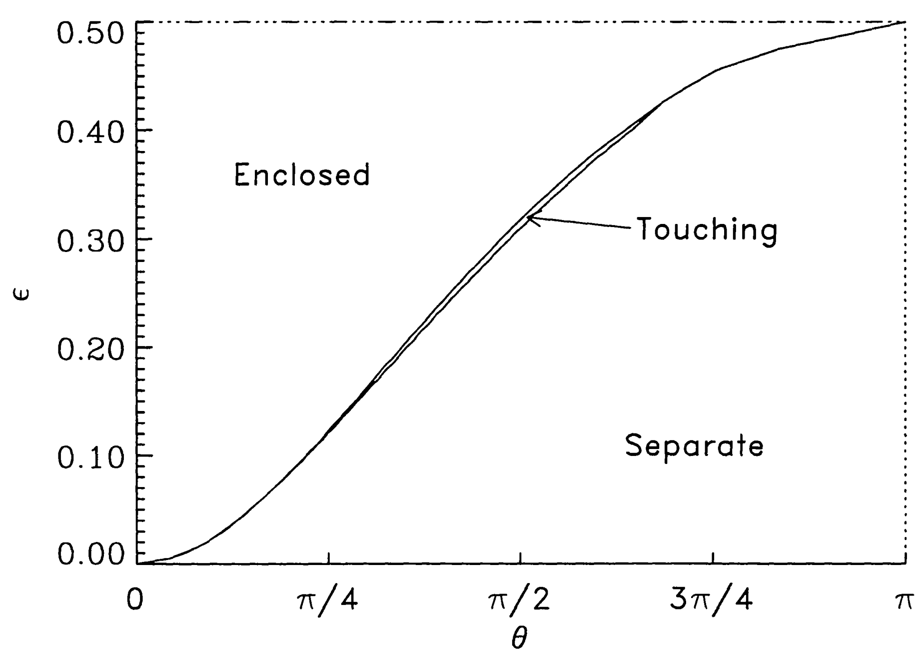


Figure 2. Bifurcation diagram for a region of parameter space with $-\epsilon = \epsilon_1 = \epsilon_2$ and $a \approx 1$ (diagram plotted for $a = 1.05$).

The topological behaviour remains generic if the parameter $\epsilon$ is introduced, where $-\epsilon = \epsilon_1 = \epsilon_2$. A bifurcation diagram in parameter space for the region of interest can then be plotted (Figure 2). This identifies the areas of parameter space where the three topological states exist and the boundaries where they bifurcate. The process of bifurcation between these states can now be analysed.

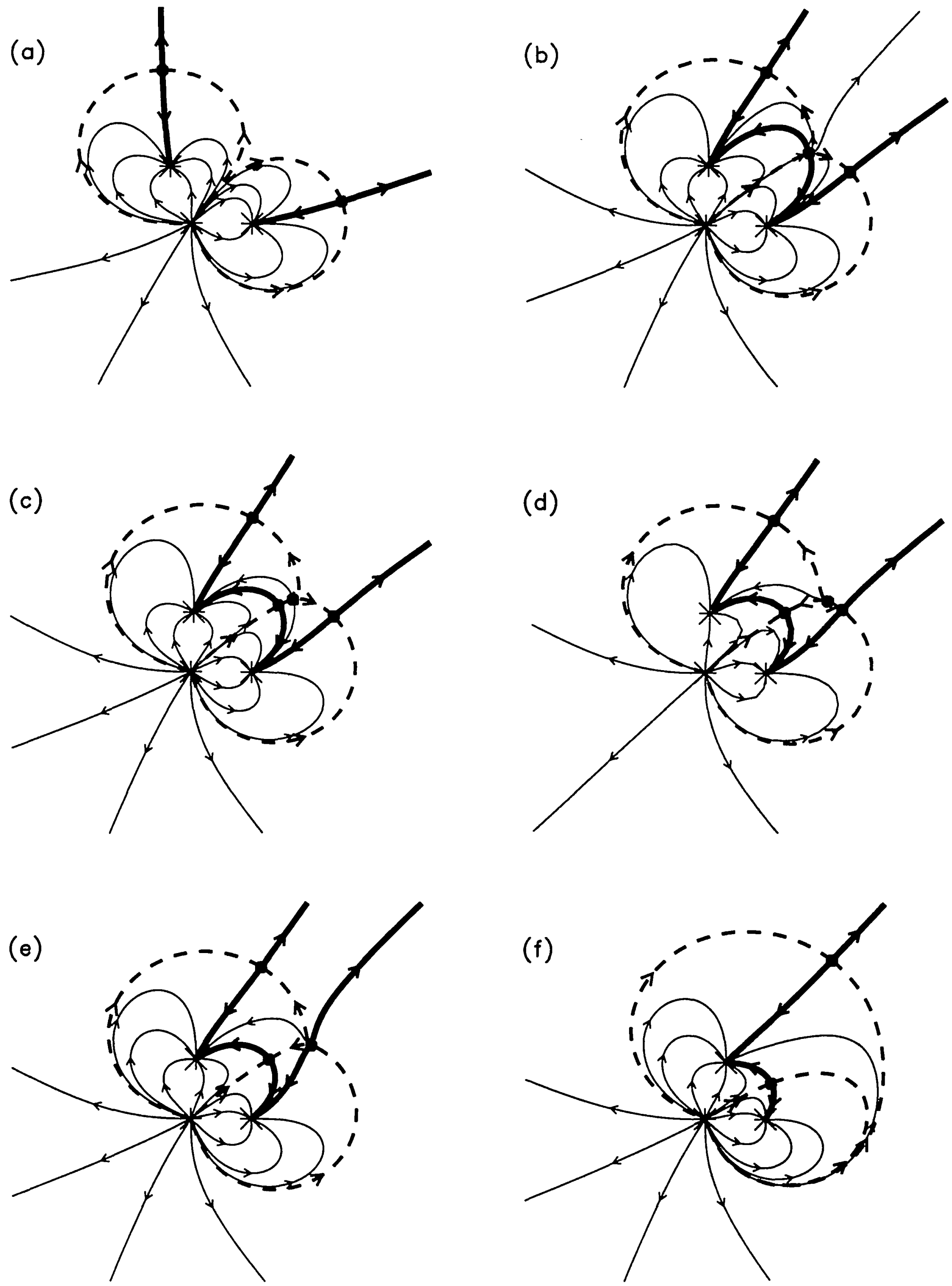


Figure 3. Local bifurcation from the separate state, (a), through the touching state, (c,d), to the enclosed state, (f), represented in the $z = 0$ plane. The stars are sources, the dots are nulls, the dashed lines are separatrices and the bold lines are spine field lines.

If the angular separation is initially wide, the system is in the separate state (Figure 3a), which has two separatrix domes apart from each other, each containing a linear null. As the angular separation decreases, the two domes become closer together until they touch (Figure 3b) forming a new second-order null at the point of touching. This state is unstable and further reduction of $\theta$ causes the second-order null to split into two linear nulls (Figure 3c). This state is stable and is known as the touching state. Further reduction of $\theta$ causes one of the new nulls to approach one of the previously existing linear nulls (Figure 3d) and the two nulls coalesce to form another unstable second-order null (Figure 3e). Finally, this null is annihilated, leaving the enclosed state with one of the separatrix domes now contained inside the other (Figure 3f).

## 4. Conclusions and Discussion

Three-source systems display a rich variety of interesting topological behaviour. There is a wide range of stable and unstable topological states, which can bifurcate, either locally or globally, between one another in a complex manner. During the course of this work we have attempted to classify all of the generic behaviour of three-source systems and much of the non-generic behaviour, as well as pinning down the regions of parameter space where the topologically stable states lie.

Active regions on the Sun may be extraordinarily complex with many different magnetic sources in the photosphere. However, the main building blocks of these complex magnetic fields are likely to be the three-source elements similar to those that have been considered here. Thus a knowledge of three-source topological behaviour allows us to develop a better understanding of the topological structure of such complex magnetic fields.

The process of bifurcation between states, due to footpoint motion or variation of flux through a source, may also offer clues to the triggering of coronal events or the formation or collapse of coronal structures. We are investigating states that are topologically representative of such structures.

It is also important to consider some higher-order cases. There is no generic state in the three-source case which allows for null points off the $z = 0$ plane, for example. However, by considering a four-source case we have been able to construct such a state and follow its evolution to study the landing of a null point on the $z = 0$ plane and its subsequent lifting back off the plane.

## References

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