Effect of nonuniform resistivity in Petschek reconnection

H. Baty\textsuperscript{a)}
Observatoire Astronomique, 11 Rue de l’Université, 67000 Strasbourg, France

E. R. Priest\textsuperscript{b)}
Institute of Mathematics, University of St. Andrews, Fife KY169SS, Scotland, United Kingdom

T. G. Forbes\textsuperscript{c)}
Institute for the Study of Earth, Oceans, and Space, University of New Hampshire, New Hampshire 03824

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The controversial topic of the existence of a Petschek solution in steady-state magnetic reconnection for a uniform resistivity is investigated, by using two-dimensional time-dependent magnetohydrodynamic simulations. A classical localized nonuniform resistivity with a profile that is exponentially decreasing from the X point is helpful to set up the Petschek solution as a first step, using a procedure with overspecified boundary conditions. The response to changing the resistivity profile in various ways is then studied. It is found that a Petschek configuration is obtained when a quasuniform resistivity is adopted. This is the case if the resistivity profile exhibits a weak negative gradient close to the X point, which dominates an inherent numerical contribution; otherwise an instability develops which disrupts the configuration. A truly uniform resistivity is probably only marginally stable. Finally, the validity of the arguments given by Kulsrud [Earth Planets Space 53, 417 (2001)] about the “incorrectness” of Petschek’s original model is discussed. © 2006 American Institute of Physics. [DOI: 10.1063/1.2172543]

I. MOTIVATION

Magnetic reconnection plays a crucial role in space and astrophysical plasmas. Indeed, it often determines the magnetic field structure and allows the conversion of magnetic energy into kinetic and thermal energies. Historically, an investigation of the reconnection process was motivated by the need to explain the solar flare phenomenon, which is known to occur on a fast time scale with impulsive energy release over typically 100 s.

Steady-state solutions in two-dimensional (2D) magnetohydrodynamics (MHD) can now be said to be fairly well established and mostly understood. The classical Sweet-Parker model is known to be too slow for a flare,\textsuperscript{1, 2} since it has a reconnection rate,

\[
M_e = R_{me}^{-1/2},
\]

where \(M_e\) and \(R_{me}\) are the Alfvén Mach and Reynolds numbers of the plasma flowing into the current sheet, respectively. In order to speed up the Sweet-Parker rate, Petschek\textsuperscript{3} proposed a fast-regime solution having a maximum reconnection rate,

\[
M_p = \pi/(8 \ln R_{mi}),
\]

which is weakly dependent on the Reynolds number. The latter rate leads in practice to a value \(M_p \approx 10^{-1} - 10^{-2}\) (since in astrophysical plasmas \(R_{me}\) is large and is typically \(10^6\) to \(10^{12}\)), which is many orders of magnitude faster than the Sweet-Parker rate. Such a high value is fast enough to explain the rapid release of magnetic energy which occurs in the solar flares. The Petschek model is based on the existence of an X-point configuration (see Fig. 1), with four slow-mode shocks radiating outward from a small central Sweet-Parker region (where a relation \(M_e = R_{mi}^{-1/2}\) holds, \(M_i\) and \(R_{mi}\) being now the Alfvén Mach and Reynolds numbers of the plasma at the entrance of the diffusion region). In the latter model, the energy conversion mostly comes from the shocks that accelerate the plasma in the form of two hot outflow jets, rather than from the central Sweet-Parker region. Later, Priest and Forbes\textsuperscript{4} discovered a more general family of steady-state solutions with the Petschek model as a particular solution.

However, there is one controversial aspect that, still now, remains puzzling (see review in the book by Priest and Forbes\textsuperscript{5}), and that could be of great importance for understanding general aspects of the reconnection mechanism in two and three dimensions. It is that numerical MHD simulations have failed to obtain a Petschek-like configuration when they use a spatially uniform resistivity.\textsuperscript{6–8} In contrast, when a nonuniform y-dependent resistivity (highly localized at the X point of the central diffusion region) is adopted, they can produce Petschek reconnection. Historically, this led to the suggestion that the Petschek model may be incorrect for a uniform resistivity,\textsuperscript{9, 10} whereas no assumption about the spatial dependence of the resistivity was made in the original model of Petschek.\textsuperscript{3} Moreover, it should be noted that in an analytical version of Petschek’s model, it has been found to be technically possible to match the resistive internal solution to the ideal external one by using average properties of the diffusion region.\textsuperscript{11}

In this article, we present careful numerical experiments, which demonstrate that it is indeed possible to establish and maintain a fast steady-state Petschek-like solution with a
the central current sheet. Typically, we are able to achieve a minimum grid spacing of $\Delta x=8 \times 10^{-4}$ and $\Delta y=3 \times 10^{-3}$ in the $x$ and $y$ directions, respectively. Additionally, for several runs, $100 \times 300$ and $100 \times 600$ spatial grid points are used to investigate the effect of using a uniform spacing in the $y$ direction. The boundary conditions are imposed through the use of two ghost cells situated slightly outside the computational domain at each boundary. We use the general finite-volume based Versatile Advection Code (VAC, see http://www.phys.uu.nl/~toth). We select the explicit one-step total variation diminishing (TVD) scheme with minmod limiting.\textsuperscript{13,14} This is a second-order accurate shock-capturing method making use of a Roe-type approximate Riemann solver. To handle the solenoidal constraint on the magnetic field $\nabla \cdot \mathbf{B}=0$, our VAC simulations apply a projection scheme at every time step in order to remove any numerically generated divergence of the magnetic field up to a pre-defined accuracy.$^{15}$

Different resistivity profiles are applied in this work. However, as a first step, all numerical experiments are started with a classical spatial form,

$$\eta(x,y) = (\eta_0 - \eta_1)|\exp[-(x/l_x)^2 - (y/l_y)^2]| + \eta_1,$$

in order to initiate the reconnection. $\eta_0$ and $\eta_1$ are the resistivities at the center of the diffusion region and in the external region, respectively. The characteristic length scales of the resistivity variation ($l_x$ and $l_y$), as well as the magnitude of the resistivity (defined by $\eta_0$ and $\eta_1$) vary from case to case.

III. RESULTS

Petschek reconnection is a particular steady-state solution of the MHD equations which requires special conditions to be met.\textsuperscript{4,16} As the latter authors pointed out, it corresponds to an undriven process that cannot easily be obtained numerically by driving a flow with external forcing. On the other hand, the use of a locally enhanced resistivity (in the current sheet) with free inflow/outflow conditions can produce a Petschek-like configuration, which is in a quasisteady state.\textsuperscript{5} An efficient strategy has been employed by Yan et al.\textsuperscript{8} in order to set up such a steady state. Indeed, in their work, the characteristic length of the nonuniform resistivity region is tailored to match the required Petschek solution (with prescribed relations between inflow and outflow conditions deduced from the analytical work of Priest and Forbes\textsuperscript{4}). A simpler alternative procedure is used in the present study.

A. Petschek solution for a localized resistivity

Indeed, we have discovered that the easiest numerical procedure to obtain a true steady-state Petschek configuration is to overspecify the system, using fixed conditions at the inflow boundary $x=L_x$; namely, imposing the mass density, two components of the flow velocity, the $y$ component of the magnetic field, and the total energy density to be fixed in time and equal to their initial values. Additionally, free conditions are imposed at the outflow boundary $y=L_y$. In this way, the system is able to choose its own inflow velocity $V_x$ (and corresponding dimensionless reconnection rate $M_x$),

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{petschek.png}
\caption{Petschek configuration. A small central diffusion region with a Sweet-Parker current sheet is extended outwards by two pairs of standing slow-mode shocks. The structure of the magnetic field lines and the direction of the inflow/outflow plasma is indicated.}
\end{figure}

quasiuniform resistivity. The term “quasiuniform” is clarified in the following. To do so, we carry out calculations using a nonlinear resistive MHD code, in the presence of well-chosen boundary conditions, which are required to allow the time-dependent solutions to follow the different regimes predicted by the analytical steady-state configurations derived by Priest and Forbes.\textsuperscript{4} Moreover, we also use our numerical results to discuss the validity of the arguments given by Kulsrud.\textsuperscript{10}

II. MODEL

The initial setup of the experiments follows previous papers,\textsuperscript{6–8} in which one assumes a current sheet structure along the $y$ axis with half-width $a$. It corresponds to an initial magnetic field parallel to the $y$ axis and varying along the $x$ direction,

$$\mathbf{B} = B_0 \tanh(x/a)y,$$

which is in static equilibrium in an isothermal medium. We set the ratio of specific heats $\gamma$ equal to $5/3$, and choose units such that the magnetic permeability is one. We set $B_0=1$, and $a=0.1$, to define our normalization. A moderate value at the boundary $x=\pm L_x$ of the plasma $\beta=0.35$ is also taken. The initial plasma pressure and density are then set to be $p(y)=1.25-B_0^2/2$ and $\rho(y)=2p(y)/\beta$, respectively. Specific inflow/outflow boundary conditions are applied at $x=\pm L_x$ and $y=\pm L_y$, respectively (see the following).

We solve the usual full set of nonlinear resistive compressible MHD equations (see Scholer,\textsuperscript{6} for example) as an initial value problem, in two spatial dimensions and Cartesian geometry $(x,y)$. In the simulations presented in this work, mainly in order to save CPU memory, we consider only one quadrant of the whole physical domain, thus considering only positive values of $x$ and $y$, $0\leq x \leq L$, and $0 \leq y \leq L_y$. Relevant symmetry boundary conditions are consequently specified along $y=0$ and $x=0$. In most runs, a total of $100 \times 100$ spatial grid points is used in the quadrant (of dimensions $L_x=1$ and $L_y=2$), with a nonuniform spacing having minimum values close to the $X$ point (at $x=y=0$). Depending on the physical case, the grid accumulation is carefully chosen in order to have sufficient cells to resolve the central current sheet.
which is different from the value $V'_e$ imposed at $x=L_x$. This is illustrated in Fig. 2, where one can see the $V_x$ profiles for a $y=0$ cut of the steady-state structures obtained in three simulations using $V'_e=−0.02$, $−0.04$, and $−0.06$. We have checked that the detailed internal structure (magnetic and flow fields) of the system is very similar in the three simulations with the same inflow velocity $V'_e=−0.036$, and the same plasma velocity at the entrance of the diffusion region $V_f=−0.042$. Thus, the overspecification allows a valid solution for the interior domain ($x⩽L_x$), although a numerical boundary layer forms between the last interior cell and the first ghost cell (situated slightly outside $L_x$). A similar procedure has been employed by Jin and Ip\textsuperscript{17} to numerically investigate some aspects of the driven reconnection process. One can also refer to Forbes and Priest\textsuperscript{18} for a general discussion on the comparison between boundary conditions in analytical and numerical models.

We have also checked that the structures obtained in this way exhibit all the features of a Petschek solution, as described in Yan \textit{et al.}\textsuperscript{8} Indeed, as the plasma flows toward the diffusion region, the streamlines converge and the magnitude of the magnetic field decreases, as expected. Figure 3 illustrates the corresponding magnetic field and current density structures. This case has been obtained by using a resistivity localized near the $X$ point (with $\eta_0=3×10^{-4}$), and exponentially decreasing toward a value $\eta_1=10^{-5}$ in the external region (which dominates the estimated residual numerical resistivity for the spatial resolution used). We have also taken $l_x=0.05$ and $l_y=0.1$, using the classical profile given by Eq. (4).

B. Petschek solution for a quasiuniform resistivity

Previous experiments\textsuperscript{6,8} have shown that the $l_y$ parameter does not influence the final Petschek configuration. This is not the case for the $l_x$ parameter, which increases the length $l$ of the central diffusion region as $l_y$ increases. Ultimately, when $l_y$ is of order of the dimensions of the whole domain (i.e., $L_y$), one obtains a Sweet-Parker structure with no slow-mode shock structures, since the diffusion region now occupies the whole length of the boundary between the opposing fields (see Fig. 8 in Scholer\textsuperscript{6} and Erkaev \textit{et al.}\textsuperscript{19}). Since increasing $l_y$ also makes the resistivity more uniform (for fixed dimensions of the domain), this led to the suggestion that the Petschek solution cannot be obtained when one uses a uniform resistivity. We have studied this, by performing a few calculations with $\eta_0=10^{-3}$, $\eta_1=10^{-5}$, and varying the $l_y$ parameter in the range $[0.03:0.35]$. The results are plotted in Fig. 4, showing the effective length $l$ (different from the imposed $l_x$, and deduced from the locus where the maximum current density has decreased by a factor of 2 in the $x=0$ direction) as a function of the measured external Mach number $M_e$, and the corresponding dependence on the internal Mach number $M_i(M_e)$. These variations agree well with the theoretical relations derived by Priest and Forbes\textsuperscript{4} for $M_i(M_e)$ (see also Eq. 10, and Fig. 6 in Forbes\textsuperscript{16}). One must note that only a part of the whole curve $M_i(M_e)$ is obtained in this way, and the maximum giving the expected maximum growth rate of $M_e=0.04$ [using Eq. (2), for a Reynolds number $Ran=L_y/\eta_0=20,000$] is not fully reached. The $l$ dependence also agrees very well with the scaling law $l\sim M_e^{-2.4}$ in Yan \textit{et al.}\textsuperscript{8} Finally, note that the Sweet-Parker solution (expected for high $l$ values) is not fully reached for the present Reynolds number, because the current sheet becomes unstable when $l⩾0.6$, as a consequence of the development of a tearing mode when the aspect ratio $H/\delta$ is of the order of $100$.\textsuperscript{8} Indeed, the width of the diffusion region follows a scaling law $\delta\sim M_e^{-1.1}$, which is more weakly dependent on the Reynolds number than $l$.

In this work, we also explore the effect of the $\eta_1$ parameter, by running different experiments for various $\eta_1$ values ranging from $\eta_1=2×10^{-5}$ to $9×10^{-5}$. The starting point of these simulations is a Petschek solution, previously obtained with a localized resistivity (i.e., using $\eta_0=10^{-4}$, $\eta_1=10^{-5}$, and $l_y=0.1$), which is subsequently evolved in time using the same resistivity profile [given by Eq. (4)] but for new $\eta_1$ values. The results are also plotted in Fig. 4. Note that, the $l$ values remain close to the curve obtained previously, except for two $\eta_1$ values (namely, $\eta_1=7×10^{-5}$ and $9×10^{-5}$). The measures for relatively high $\eta_1$ are also probably influenced by the smoothing of the shock structures that are thicker in the presence of higher dissipation, as one can deduce from...
and 9/H20849 from 0.03 to 0.3. The stars are obtained from runs investigating the internal Mach number M/H9257 threshold of a gradual increase of even more weakly nonuniform resistivity, by using a more

In fact, it is possible to achieve a final steady-state with an effect due to the grid accumulation near the X point. Thus, it seems that an explicit negative difference or gradient determined by η0 − η1 must dominate the inherent numerical resistivity gradient in order to maintain a Petschek solution.

Next, following the same procedure used for the study of the η1 effect, we have run cases using another form of resistivity profile in the subsequent time evolution (i.e., starting from a Petschek solution obtained by using a localized resistivity), which has a y cosine variation,

$$\eta(y) = \eta_0 (1 + \epsilon \cos(2\pi y/H_y)).$$

FIG. 4. Current density obtained for η1 = 3 × 10^{-5}, 7 × 10^{-5}, and 9 × 10^{-5} (from left to right, respectively).

FIG. 5. Current density obtained for η1 = 3 × 10^{-5}, 7 × 10^{-5}, and 9 × 10^{-5} (from left to right, respectively). We take H_y values that are small compared to l, and ε values much smaller than unity. In this way, the resistivity is uniform on average in the diffusion region, with a weak chosen nonuniformity given by the amplitude ε. With this form, we have been able to obtain a steady-state Petschek solution using η0 = 10^{-5} (as previously), as long as the parameter ε is not too small with a threshold value of order 2 × 10^{-2}. Again, this means that the weak local nonuniformity due to the spatial numerical resistivity variation must be dominated by the prescribed weak nonuniformity of Eq. (5). We have additionally shown that it is the spatial dependence close to the X point that is in fact crucial, by performing a case with the same cosine variation but localized within the diffusion region; elsewhere, a uniform resistivity profile was taken.

Finally, we have explored the effect of a positive or negative gradient of the resistivity, with a strong localization at y = y_c close to the X point. The amplitude of the gradient is small but higher than the numerical nonuniformity (see the previous discussion). This means that a uniform resistivity profile η_l/η_c is prescribed to the left and right of y = y_c. The results show that a new Petschek solution is reached when η_l/η_c is negative, whereas a disruption of the whole structure is produced for η_l/η_c positive. The disruption is very similar

FIG. 5. Length of the diffusion region l as a function of the external Mach number M_e (upper panel), and external Mach number M_e as a function of the internal Mach number M_i (lower panel). The values of l are measured as the y locus where the maximum density current has decreased by a factor of 2, in the x=0 direction. The circles are obtained from runs using a localized nonuniform resistivity [following Eq. (4)] for different η1 values ranging from 0.03 to 0.3. The stars are obtained from runs investigating the η1 effect (see the text), for different η1 values (2 × 10^{-5}, 3 × 10^{-5}, 5 × 10^{-5}, 7 × 10^{-5}, and 9 × 10^{-5}).
to cases where a uniform resistivity is prescribed everywhere. The analysis of the time evolution of the $B_y(y)$ profile for the $x=0$ axis can give useful information. Indeed, as shown in Fig. 6, a positive value $B_y(y)$ is maintained for a negative resistivity gradient, which seems to lead to constant $B_y(y)$ for $y \ll y_c$ in time. Conversely, for a positive resistivity gradient, $B_y(y)$ becomes negative and grows in amplitude with time, penetrating the external region (see Fig. 7). As a positive $B_y$ is required to sustain a shock structure, one finds that the structure is unstable if $B_y(y)$ reverses somewhere in the diffusion region. Note that, for the stable case, the new Petschek solution that is obtained exhibits a $B_x$ value that is slightly smaller than the initial one, that is $B_n=0.19$ compared to $B_n=0.23$. This is not surprising as the resistivity gradient introduced in this way determines a new $B_x(y)$ structure in the diffusion region.

Anyway, in all these experiments with a quasiuniform resistivity, a Petschek solution is obtained. Its rate is relatively far from the maximum rate. This is due to the fact that, in one way or another, when a gradient is numerically introduced somewhere, an effective characteristic length is also introduced that consequently determines the rate.

C. Kulsrud arguments

Kulsrud\cite{Kulsrud:2006} claimed to show that Petschek’s mechanism is “incorrect” when the magnetic diffusivity is uniform, so let us examine his arguments in detail.

First of all, he suggests in the introduction that the discrepancy between the Sweet-Parker timescale and the observed energy-release timescale (of say, 100 s) in a solar flare could be solved by adopting an anomalous diffusivity. But a typical flare Alfvén time of 100 s and classical magnetic Reynolds number of $10^8$ would need an anomalous diffusivity (due to, say, ion acoustic instability) to be enhanced by a factor of $10^6$, which is most unlikely. A much more natural explanation for the flare time scale is fast reconnection (with presumably a diffusivity locally enhanced by a small amount) driven by eruptive instability (see, e.g., Priest and Forbes\cite{Priest:1992}).

Later in the introduction, he suggests that the controversy between the Sweet-Parker and Petschek mechanisms seemed to be settled by Biskamp’s numerical simulations. This is a misunderstanding of the implications of Biskamp’s very impressive work. As has been detailed by Priest and Forbes\cite{Priest:1992}, Petschek’s model is one family of almost-uniform reconnection regimes, in which the inflow region is a perturbation to a uniform magnetic field: the different members of the family have different inflow boundary conditions, and all need a slightly enhanced diffusivity in the diffusion region. A completely different reconnection solution, namely nonuniform reconnection, was proposed by Priest and Lee\cite{Priest:1992}, in which the inflow regime is highly nonuniform and is a perturbation to an X-point field. This was later generalized by Strachan and Priest\cite{Strachan:2000} to give a family of nonuniform reconnection regimes with pressure gradients and different inflow boundary conditions, and with a scaling that is similar to Sweet-Parker one. Now, Biskamp’s numerical experiment on reconnection was the best of the time and found long (non-Petschek-like) diffusion regions with slow reconnection and a quasi-Sweet-Parker scaling; so he concluded that fast reconnection and Petschek reconnection does not exist. However, the boundary and initial conditions that Biskamp imposed do not correspond to those necessary for Petschek reconnection, but rather are those appropriate to nonuniform reconnection and in fact lead to the long diffusion region and exactly the same scaling as found in nonuniform reconnection. Thus, the significance of Biskamp’s experiments is not to imply that Petschek reconnection is incorrect but rather to demonstrate that a wider range of reconnection solutions exists than Petschek had envisaged.

Kulsrud’s paper has several misconceptions about reconnection theory. He suggests that the flow speed is everywhere much smaller than the Alfvén speed outside the reconnection layer. Although this is true for the inflow region, it is false for the outflow, which is Alfvénic and interacts with the surrounding medium, often leading to a fast shock downstream where it is slowed down. He also suggests that the diffusion region in Petschek’s mechanism differs from a Sweet-Parker
diffusion region in that acceleration along the region is by magnetic tension in the first case and by plasma pressure in the second. This distinction is false and has been discussed at length by Priest and Forbes, who show how in both cases the magnetic force accelerates the plasma; if the outflow plasma pressure differs from the plasma pressure at the null point, then a pressure gradient also plays a role, leading to an outflow velocity different from the Alfvén speed. For a pure Sweet-Parker layer, acceleration is by the magnetic force alone and the outflow is exactly Alfvénic.

Kulsrud suggests that the length of the diffusion region \( l \) is a free parameter in Petschek theory, but that is simply not true. In Petschek’s mechanism a whole range of inflow speeds (or reconnection rates) at large distances is allowed, varying between the Sweet-Parker value and the well-known maximum value that scales with the inverse logarithm of the Reynolds number [see Eq. (2)]. The diffusion region length is determined by the distant inflow speed, in such a way that, as the reconnection rate increases, so the shock angle increases and the diffusion region size decreases until the maximum rate is reached.

The crux of Kulsrud’s criticism, however, is his suggestion that (since \( l \) is a free parameter) Petschek left out one condition that must be satisfied and that implies that \( l \) must equal the external macroscale \( L_y \). He proposes a physical idea and a new equation, which he suggests Petschek overlooks, as follows. The idea is that the magnetic field must be regenerated as it is swept away along the diffusion region and that the regeneration is due to the rotation of field lines as they enter the diffusion region. However, as elegantly shown by Hornig, in two-dimensional reconnection, magnetic field lines break at the \( X \) point but magnetic flux is naturally conserved as it enters and leaves the diffusion region along the axes of symmetry according to the induction equation. Indeed, in a steady-state the presence of the transverse field component \( B_y \) is a natural consequence of flux conservation: thus the fact that the electric field \( E \) is uniform determines the magnetic field on the \( y \) axis from

\[
E - V_x B_y = - \eta \frac{\partial B_y}{\partial y} \tag{6}
\]

in terms of the flow speed. This leads to the relationship \( V_x B_y = V_y B_o \) between the input \( (V_x, B_i) \) and output \( (V_y, B_o) \) flow speed and field strength, which is indeed included in Sweet-Parker and Petschek theory. The new heuristic equation (for a balance between merging and down-sweeping) proposed by Kulsrud is,

\[
B_y \frac{d \theta}{dt} - \frac{B_x V_A}{l} = 0, \tag{7}
\]

where the first term represents a regeneration (the effect on the \( y \) component of the magnetic field of a rotation \( d \theta / dt \)) and the second term represents the transport along the layer of the transverse field component \( (B_y) \). \( V_A \) is the local Alfvén speed. However, we have checked whether this equation holds in our simulations (with a quasiuniform resistivity) and find that the first term is very small and is one order of magnitude smaller than the second one.

Kulsrud also makes two further ad hoc assumptions before being able to deduce from it that \( l \approx L_y \) (for uniform resistivity) so that Petschek’s solution fails. His first assumption is that \( V_x / B_0 \) constant so that the speed \( V_x \) with which each field lines enters the diffusion layer is proportional to \( B \). In our simulations, we find no evidence of this relation as one can see in Fig. 8. The second ad hoc assumption is that the external field \( B_y \) along the edge of the diffusion region varies as \( B_y = B_0 (1 - y^2 / L_y^2) \). However, it is not true that in Petschek theory the nonuniformity of the external longitudinal field is on the overall external scale \( L_y \) along the diffusion region. Rather, it is the solution of Laplace’s equation in the external region with the transverse field component \( (B_y) \) as a boundary condition, varying with a scale \( l \) along the diffusion region, as also indicated by our numerical experiments (see Fig. 9). We therefore conclude on both mathematical, physical and numerical grounds that Kulsrud’s criticism of the Petschek mechanism is based on Eq. (7) is unfounded.
IV. SUMMARY

We have discovered an easy method to obtain numerically a Petschek solution by overspecifying the boundary conditions. The usual exponentially decreasing resistivity profile helps to set up the solution in a first stage, in agreement with previous experiments, because it explicitly introduces a characteristic length \( l \) for the diffusion region via an \( l \) parameter. The solution \( M_0(M_e) \) follows. We can maintain a Petschek solution by making this profile more uniform (via the increase of \( \eta \), the background resistivity). However, an exactly uniform profile cannot be used in such numerical experiments, since inherent numerical gradients are unavoidable. A quasuniform resistivity profile having a small negative gradient (in the \( y \) direction) and dominating an inherent numerical contribution close to the \( x \) point is necessary for a stable Petschek configuration, otherwise an instability disrupts the conﬁguration. Using a cosine variation for the resistivity can also produce a Petschek solution in a similar way. A truly uniform resistivity configuration is thus probably only marginally stable. It is not possible with a quasiuniform resistivity proﬁle having a small negative gradient. A nonuniform resistivity proﬁle to obtain a solution with a rate \( M_e \) close to its maximum value, because in numerical experiments one implicitly introduces a characteristic length \( l \) that is not optimal.

We conclude that Petschek’s model and other fast reconnection regimes are valid when the resistivity is enhanced in the diffusion region or is close to uniform. Indeed, one ripple of a resistivity fluctuation of small amplitude is suﬃcient to seed the Petschek mode. Statements that it is invalid because of improper matching between the diffusion region and the external region are not correct. When the resistivity is exactly uniform, Petschek’s mechanism is likely to be unstable due to some process that has not been identified. However, from a practical point of view, one would not expect the resistivity to be precisely uniform. A nonuniform resistivity appears to be necessary for two reasons. First, it is needed in order to set up a Petschek-type conﬁguration starting from a current sheet with a field proﬁle of a hyperbolic tangent (with boundary conditions consistent with Petschek’s solution). Second, a small negative resistivity gradient near the null point is necessary to maintain the numerical stability of the solution. This gradient also inﬂuences the exact solution in a way that remains to be explored in detail in the future. Finally, one must note that this result is not a pure academic point (as sometimes suggested in the literature), since we show that Kulsrud’s argument for the “incorrectness” of Petschek’s mechanism is itself flawed, as his proposed extra heuristic equation is invalid. We conclude that Petschek’s solution and other fast regimes are likely to occur in practice in a fully collisional plasma. This is important for a more general understanding of reconnection in three dimensions.

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