Magnetohydrodynamic Waves

Magnetohydrodynamic waves are found in a wide variety of astrophysical plasmas. They have been measured in plasma fusion devices and detected in the magnetosphere of Earth, the solar wind and a number of magnetic structures seen in the Sun’s atmosphere. In space plasmas their detection is often indirect, by matching measured properties (such as propagation speed or pressure variation) with theoretically deduced properties and correlations, although the magnetosphere and solar wind provide direct in situ measurements. A complication in the identification of magnetohydrodynamic waves in space plasmas has been the realization that generally such plasmas are strongly inhomogeneous, structured by magnetism, plasma density and temperature variations, or by plasma flows. This strong inhomogeneity brings about complications in the description of magnetohydrodynamic wave phenomena, with magnetic flux tubes and plasma flow tubes being of special interest. In the solar atmosphere, sunspots, solar photospheric magnetic flux tubes, coronal loops and polar plumes are examples of flux tube structure, and oscillatory phenomena have been detected in all of them. Oscillations have also been detected in solar prominences and generated by solar flares.

Waves are the way in which information in a medium may be communicated from one region to another. In a gas communication can be achieved by sound waves, in the form of compressions and rarefactions of the medium which propagate at the sound speed $c_s$. For an ideal gas with equilibrium pressure $p_0$ and density $\rho_0$ the sound speed in a medium is given by

$$c_s = \left( \frac{\gamma p_0}{\rho_0} \right)^{1/2}$$

where $\gamma$ is the ratio of specific heats. Magnetism introduces an additional speed, determined by the strength $B_0$ of the magnetic field and the plasma density $\rho_0$. This so-called Alfvén speed $v_A$ is defined by

$$v_A = \left( \frac{B_0^2}{\mu \rho_0} \right)^{1/2}$$

where $\mu$ is the magnetic permeability of the medium. These two speeds determine the nature of wave propagation in magnetohydrodynamics.

The speeds $c_s$ and $v_A$ that characterize sound and magnetic effects are conveniently supplemented by the introduction of two further speeds, a slow speed $c_t$ and a fast speed $c_f$, defined by

$$c_t^2 = c_s^2 + v_A^2, \quad c_f^2 = c_s^2 + v_A^2.$$

The slow speed is evidently both sub-sonic and sub-Alfvénic; the fast speed $c_f$ is super-sonic and super-Alfvénic. The slow speed $c_t$ proves to be of particular significance in magnetic flux tubes and is accordingly sometimes referred to as the tube speed, although it is not the only speed of special significance in flux tubes (see below). The four speeds $c_t, c_s, v_A$ and $c_f$ arise in various ways in the description of magnetohydrodynamic wave phenomena, and their magnitudes may vary dramatically from application to application. For example, in a $2 \times 10^5$ K solar corona with a magnetic field strength of $10^2$ G the sound speed is about $200$ km s$^{-1}$ and the Alfvén speed about $10^2$ km s$^{-1}$, whereas in a solar photospheric flux tube both these speeds fall to about $10$ km s$^{-1}$. The corresponding slow and fast speeds in the solar coronal illustration are $196$ km s$^{-1}$ and $1020$ km s$^{-1}$, whereas in the photospheric flux tube these speeds are $7$ km s$^{-1}$ and $14$ km s$^{-1}$, respectively.

The distinctive feature of a magnetohydrodynamic plasma is that, in addition to compressive disturbances, the magnetoacoustic waves with their affinity to sound waves, there arises the possibility of a wave which propagates without compressing the plasma, driven purely by magnetic tension forces. This is the Alfvén wave, named in honour of the 1970 Nobel Laureate Hannes Alfvén (1908–1995) who first described the wave in 1942 and made fundamental contributions to the foundations of magnetohydrodynamics. Alfvén waves are closely analogous to the familiar transverse waves of an elastic string, the magnetic field providing the tension force that determines (together with the string or plasma density) the speed of propagation of the wave. The magnetic field in a plasma is also able to provide another force, akin to the plasma pressure; this is the magnetic pressure, $B_0^2/(2\mu)$ N m$^{-2}$. Overall the field provides magnetic pressure forces that resist any tendency to compress the plasma (acting in addition to the plasma pressure) and magnetic tension forces which act to straighten any distortions in the magnetic field. The tension force gives rise to the Alfvén wave; the magnetic pressure force gives rise to slow and fast magnetoacoustic waves. Broadly, the fast magnetoacoustic wave results when the magnetic pressure force acts in unison with the plasma pressure force; the slow magnetoacoustic wave results when these two pressure forces act in opposition.

To describe magnetohydrodynamic waves in detail it is necessary to examine the equations of ideal magnetohydrodynamics. We consider an inhomogeneous medium because it allows a description of waves in the important case of a magnetic flux tube but also includes, as a special case, the classical problem of waves in a uniform medium. Consider, then, a basic state in which there is an applied unidirectional magnetic field $B_0 = B_0 \hat{z}$ aligned with the $z$-axis in an appropriate coordinate system. In equilibrium the plasma density is $\rho_0$, the temperature $T_0$, the pressure $p_0$ and the field strength $B_0$. These quantities may all vary in a direction perpendicular to the applied magnetic field; equilibrium is achieved by requiring that

$$\text{grad} \left( \rho_0 + \frac{B_0^2}{2\mu} \right) = 0$$

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Magnetohydrodynamics yield the four speeds along the field. To examine small-amplitude linear disturbances about this equilibrium, it is convenient to introduce the perturbation in total pressure, \( p_T \):

\[
p_T = p + \frac{1}{\mu} B_B B.
\]

Then, splitting the velocity field \( v \) into components perpendicular and parallel to the applied magnetic field, so that \( v = v_z + v_{\perp} \), the linearized equations of ideal magnetohydrodynamics yield

\[
\rho_0 \left( \frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial z^2} \right) v_z + \nabla_\perp \left( \frac{\partial p_T}{\partial t} \right) = 0
\]

\[
\rho_0 \left( \frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial z^2} \right) v_{\perp} + \frac{c_s^2}{c_A^2} \frac{\partial^2}{\partial t^2} v_{\perp} = 0
\]

\[
\frac{\partial p_T}{\partial t} = \rho_0 v_z \frac{\partial v_z}{\partial z} - \rho_0 c_s^2 \text{div} \, v.
\]

This system of coupled partial differential equations determines the relationship between the flow and the total pressure variation \( p_T \). The operator \( \nabla_\perp \) denotes the component of the gradient operator that acts perpendicular to the applied magnetic field (i.e., perpendicular to the \( z \)-axis). Notice the occurrence of the four speeds \( c_s, c_A, v_A \) and \( c_\perp \), each a function of the coordinates perpendicular to the applied magnetic field.

It may be noted that equations (7) and (8) combine to give

\[
\frac{\partial^4 v_z}{\partial t^4} = c_s^2 \frac{\partial^2}{\partial z^2} (\text{div} \, v).
\]

The absence of magnetic effects in this equation is simply a reflection of the fact that magnetic forces act perpendicular to the magnetic field and so have no effect on motions along the field.

Uniform medium

In the case of a uniform medium, \( B_0, p_0, T_0 \) and \( \rho_0 \) are all constants and so are the basic speeds \( c_s, c_A, v_A \) and \( c_\perp \). This allows a relatively straightforward reduction of equations (6)-(8). It proves convenient to work in terms of the compressibility \( \Delta \equiv \text{div} \, v \). Then \( \Delta \) satisfies the fourth-order wave equation

\[
\frac{\partial^4 \Delta}{\partial t^4} - (c_s^2 + v_\perp^2) \frac{\partial^2}{\partial t^2} \nabla^2 \Delta + c_s^2 v_A^2 \frac{\partial^2}{\partial z^2} \nabla^2 \Delta = 0
\]

involving the three-dimensional Laplacian \( \nabla^2 \).

Since the medium is uniform and unbounded we may Fourier analyse equation (10) in a Cartesian coordinate system (with position vector \( r = (x, y, z) \)), writing

\[
\Delta(x, y, z, t) = \Delta_0 \exp i(\omega t - k \cdot r)
\]

where \( \Delta_0 \) is the amplitude (a constant), \( \omega \) the frequency and \( k = (k_x, k_y, k_z) \) the wavevector of the disturbance; other variables (e.g. flow \( v \) and field \( B \)) are represented similarly. The Laplacian operator becomes

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = -k^2
\]

where \( k = (k_x^2 + k_y^2 + k_z^2)^{1/2} \) is the magnitude of the wavevector. Then, for \( \Delta_0 \neq 0 \), equation (10) yields

\[
\omega^2 - (c_s^2 + v_\perp^2) k^2 \omega^2 + c_s^2 v_A^2 k^2 k^2 = 0.
\]

This is the dispersion relation for magnetoacoustic waves; it relates the wave frequency \( \omega \) to the wavevector \( k \), thus describing the propagation.

The dispersion relation (12) may be solved to give the phase speed \( c (\equiv \omega/k) \). In terms of the angle \( \phi \) that the wavevector \( k \) makes with the applied magnetic field \( B_0 \), with \( k_x = k \cos \phi \), we obtain

\[
c^2 = \frac{1}{2} c_\perp^2 \left[ 1 \pm \sqrt{1 - 4 c_s^2 \cos^2 \phi} \right]^{1/2}.
\]

The smaller root (corresponding to the \( - \) sign) is the slow magnetoacoustic wave, and the larger root (corresponding to the \( + \) sign) is the fast magnetoacoustic wave. The waves are compressive (since \( \Delta \neq 0 \) and so involve density, pressure and temperature variations—as do sound waves. Unlike sound waves, magnetoacoustic waves exhibit anisotropic propagation, their phase speeds depending on the angle of propagation to the applied magnetic field; this is simply a reflection of the directionality that an applied magnetic field imposes on a plasma. When propagation is along the applied magnetic field \( B_0 \), so that \( \phi = 0 \), then \( c^2 = c_s^2 \) or \( c_A^2 \). When propagation is perpendicular to the applied magnetic field, so that \( \phi = \pi/2 \), then \( c^2 = c_\perp^2 \). In other words, the slow wave is unable to propagate across the magnetic field, whereas the fast wave propagates with the speed \( c_f \). This is the greatest speed that the fast wave can achieve; at any other angle its speed lies below \( c_f \) and above (or equal to) the greater of \( c_s \) and \( v_A \). The phase speed of the slow wave is always less than or equal to the smaller of \( c_s \) and \( v_A \).

Magnetoacoustic waves are compressive but disturbances that are incompressible (\( \Delta = 0 \))—producing no changes in plasma density of an element moving with the fluid—are also permitted and in fact provide a distinct form of wave motion. This is the Alfvén wave with dispersion relation

\[
\omega^2 = k_0^2 v_A^2.
\]
giving a phase speed \( c \) according to
\[
   c^2 = v_A^2 \cos^2 \phi. \tag{15}
\]
Evidently, then, the phase speed of an Alfvén wave cannot exceed the Alfvén speed, \( v_A \). Moreover, like the slow wave, the Alfvén wave is unable to propagate across the magnetic field, since \( \omega^2 = 0 \) when \( \phi = \pi/2 \). The Alfvén wave involves no motions along the magnetic field (\( v_z = 0 \)) and no variations in plasma density (\( \rho = 0 \)), pressure (\( \rho = 0 \)), temperature or total pressure (\( p_t = 0 \)). Thus the Alfvén wave is a transverse wave that propagates incompressibly, involving no changes in plasma density or pressure.

Altogether, then, we see that in the presence of an applied magnetic field there are three waves available to the medium, the Alfvén wave and the fast and slow magnetoacoustic waves. All three waves exhibit a degree of anisotropic propagation but this is most marked in the Alfvén wave and the slow wave, neither of which can propagate orthogonally to the applied magnetic field; the fast wave is only weakly anisotropic and achieves its greatest phase speed (\( c = c_f \)) when propagating perpendicularly to the applied magnetic field. The anisotropy exhibited by magnetohydrodynamic waves is in sharp contrast to the isotropic behavior of sound waves in an uniform medium, this simply being a reflection of the directionality imposed by a magnetic field.

The behavior of the magnetohydrodynamic wave speeds is conveniently represented in a diagram by plotting the phase speed \( c \) as a function of propagation angle \( \phi \), using a polar coordinate \((c, \phi)\) representation; see figure 1.

Related to the information provided in the phase speed plot is the behavior of the group velocity \( c_g \) of a wave, defined by
\[
   c_g = \frac{\partial \omega}{\partial k}. \tag{16}
\]
The group velocity of a wave is the velocity with which energy (and information) is propagated by the wave. For the Alfvén wave equation (16) gives simply \( c_g = v_A \hat{z} \), showing that an Alfvén wave propagates energy and information along the applied magnetic field, whatever the direction of the wave itself. The group velocity for magnetoacoustic waves is more complicated to obtain and is usually displayed in diagrammatic form, showing a plot of the group speed \( c_g = |c_g| \) as a function of propagation angle; see figure 2. Again, note the highly anisotropic behavior of the slow wave which is confined to propagating energy close to the applied magnetic field, whereas the fast wave propagates energy approximately isotropically in all directions.

Magnetically structured media

The nonuniform media generally encountered in space plasmas may frequently be modeled in terms of a magnetically structured plasma. Structuring may be in the magnetic field, plasma density or temperature, or through the presence of plasma flows. Both Cartesian and cylindrical coordinate representations are possible. The cylindrical form is particularly interesting in view of the wide-spread occurrence of flux tube structures. Returning to equations (6)–(8), we express them in cylindrical coordinates \((r, \theta, z)\), writing \( \mathbf{v} = (v_r, v_\theta, v_z) \). Equation (6)
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then yields

\[ \rho_0(r) \left( \frac{\partial^2}{\partial t^2} - v_A^2(r) \frac{\partial^2}{\partial z^2} \right) v_\theta + \frac{\partial^2 p_T}{\partial \theta \partial t} = 0 \]  
(17)

\[ \rho_0(r) \left( \frac{\partial^2}{\partial t^2} - v_A^2(r) \frac{\partial^2}{\partial z^2} \right) v_\theta + \frac{1}{r} \frac{\partial^2 p_T}{\partial \theta \partial t} = 0. \]  
(18)

These equations may be reduced to ordinary differential equations by writing

\[ v_\theta(r, \theta, z, t) = v_\theta(r) \exp(i(\omega t + n\theta - k_z z)) \]  
(19)

for mode number \( n = 0, 1, 2, \ldots \). The resulting equations may be manipulated to yield the equation

\[ \frac{d}{dr} \left( \frac{\rho_0(r)(k_A^2 v_A^2(r) - \omega^2)}{[m^2(r) + n^2/r^2]} \right) \frac{d}{dr} (r v_\theta) = \rho_0(r)(k_A^2 v_A^2(r) - \omega^2) v_\theta \]  
(20)

for \( v_\theta \), with \( m^2 \) given by

\[ m^2(r) = \left( k_A^2 v_A^2(r) - \omega^2 \right) / \left( c_A^2 + v_A^2(k_A^2 v_A^2(r) - \omega^2) \right). \]

Equation (20) is sometimes called the Hain-Lüst equation. The differential equation satisfied by \( p_T \) is also of interest:

\[ \rho_0(r)[k_A^2 v_A^2(r) - \omega^2] \frac{1}{r} \frac{d}{dr} \left( \frac{1}{\rho_0(r)(k_A^2 v_A^2(r) - \omega^2)} \frac{d p_T}{d r} \right) = \left(m^2(r) + \frac{n^2}{r^2}\right) p_T. \]  
(21)

Equations (20) and (21) possess singularities at \( \omega^2 = k_A^2 v_A^2 \) and \( \omega^2 = k_A^2 c_A^2 \), where the leading coefficients of the differential equations vanish; these singularities generate the Alfvén continuum and the slow continuum, respectively. The presence of these singularities is an indication of a number of interesting effects connected with the phenomenon of resonant absorption, of particular interest in the question of solar CORONAL HEATING MECHANISMS and in the absorption of \( p \)-modes in sunspots.

There is a simple solution of equations (8), (17), and (18) that we may note immediately. If \( \partial / \partial \theta = 0 \), corresponding to symmetric disturbances of the tube, then equation (18) reduces to

\[ \frac{\partial^2 v_\theta}{\partial z^2} = v_A^2(r) \frac{\partial^2 v_\theta}{\partial z^2}. \]  
(22)

This is a torsional Alfvén wave which propagates torsional oscillations \( v_\theta \) with the local value of the Alfvén speed \( v_A(r) \). Equation (22) illustrates the phenomenon of phase mixing of Alfvén waves, whereby a wave rapidly becomes highly corrugated with large radial gradients building up in \( v_\theta \). The buildup of such large gradients is important for the damping of magnetohydrodynamic waves because ordinarily the Alfvén wave in particular is difficult to damp. In a uniform medium the Alfvén wave may propagate vast distances before it is significantly damped by ohmic or viscous processes. However, with phase mixing or resonant absorption operating damping is considerably more efficient.

We may illustrate phase mixing by noting that the solution

\[ v_\theta = v_\theta \sin k_z[z - v_A(r)t] \]  
(23)

of the Alfvén wave equation (22) gives a wave of amplitude \( v_\theta \), and \( |v_\theta| \leq v_\theta \) at all times. However, the radial gradient \( \partial v_\theta / \partial r \) of the motion grows in amplitude linearly in time \( t \), for

\[ \frac{\partial v_\theta}{\partial r} = -v_\theta \frac{\partial}{\partial r} k_z \cos k_z[z - v_A(r)t] \]  
(24)

where \( v_\theta \) denotes the slope of the Alfvén speed profile. Thus \( \partial v_\theta / \partial r \) grows secularly on a timescale of \( 1/v_\theta \). This phase mixing timescale can be very short. For example, in a coronal loop where the Alfvén speed changes from say 2000 km s\(^{-1}\) in the loop to 1000 km s\(^{-1}\) in the loop’s immediate environment, across a distance of 500 km, the phase mixing timescale is 0.5 s. The result of such phase mixing—and resonant absorption achieves similar results—is that wave damping may occur in significantly shorter times or distances than is the case for magnetohydrodynamic waves in a uniform medium. In fact, an estimate of wave damping for an Alfvén wave undergoing phase mixing in a medium with Alfvén speed \( v_A \) gives a damping length (the distance a wave travels before its amplitude is reduced by a factor e, or 37%, of its initial value) of order \( (6v_\theta^2/\omega^2 v_A^2)^{1/3} \). The cube root dependence means that damping lengths in a structured medium are considerably shorter than those in a uniform medium. For the coronal conditions illustrated above (and taking \( v \) to be \( 2 \times 10^{13} \) m\(^2\) s\(^{-1}\)), the damping length for a wave of period 100 s is about \( 10^5 \) km. Such considerations are important in the question of coronal heating.

**Flux tube waves**

Magnetic flux tubes, regions of magnetic field that are distinguished from one another through plasma density or temperature, magnetic field strength or plasma flow, are of particular interest. Two types of flux tube have received detailed study: the isolated flux tube of magnetic field embedded in a field-free medium and the magnetically embedded tube of dense plasma in a magnetic medium. The isolated tube finds specific application in the solar photospheric tube (see SOLAR PHOTOSPHERIC MAGNETIC FLUX TUBES) and the embedded tube has application to the solar wind and the coronal loop.

Consider again equations (20) and (21), examining the specific case of a flux tube of radius \( a \), field strength \( B_0 \) and plasma density \( \rho_0 \) embedded in a magnetic environment with field strength \( B_0 \) and plasma density \( \rho_0 \):

\[ B_0(r) = \begin{cases} B_0, & r < a \\ \rho_0, & r > a \end{cases} \]

\[ \rho_0(r) = \begin{cases} \rho_0, & r < a \\ \rho_0, & r > a. \end{cases} \]  
(25)
The case tube waves defines the geometry of the vibrating tube. The Fourier analysis in equation (19) in the description of case commonly referred to as the sausage wave (or mode). The boundary. See figure 4. Figure 3. The equilibrium state of a magnetic flux tube of radius \(a\), field strength \(B_0\) and plasma density \(n_0\) embedded in a magnetized plasma of field strength \(B_e\) and plasma density \(\rho_e\).

The tube is in magnetostatic pressure balance with its surroundings (see equation (4)). The Alfvén, sound and slow speeds within the tube are \(v_A\), \(c_s\) and \(c_t\), and their values in the external medium are \(v_{Ae}\), \(c_e\) and \(c_{te}\) (see figure 3). The case of an isolated tube embedded in a field-free environment corresponds to setting \(B_e = 0\) (so that \(v_{Ae} = 0\)), but for magnetically embedded tubes in a strongly magnetized plasma (i.e. a plasma in which the Alfvén speeds exceed the sound speeds) the field strengths \(B_0\) and \(B_e\) are generally comparable and, moreover, \(v_{Ae}, v_A > c_s, c_t\).

For the equilibrium (25) the differential equation (21) may be solved for \(p_r\) in terms of Bessel functions \(J_n(\alpha r)\) in \(r < a\) and \(K_n(\alpha r)\) in \(r > a\), with the result

\[
\rho_e(k_e^2v_{ Ae}^2 - \omega^2)n_n(\alpha a) = \rho_0(k^2v_A^2 - \omega^2)m_n(\alpha a)K_n(m_n(\alpha a)K_n(m_n(\alpha a)) (26)
\]

where

\[
n_n^2 = \frac{(\omega^2 - k_e^2v_{ Ae}^2)(\omega^2 - k_e^2c_e^2)}{(c_s^2 + \omega^2)(\omega^2 - k_e^2c_e^2)}
\]

and

\[
m_n^2 = \frac{(k_e^2c_e^2 - \omega^2)(k_e^2v_{ Ae}^2 - \omega^2)}{(c_s^2 + \omega^2)(k_e^2v_{ Ae}^2 - \omega^2)}.
\]

Equation (26) is the dispersion relation for flux tube waves, describing magnetoacoustic waves in a uniform magnetic flux tube; it applies for waves that are confined to the tube, requiring that \(m_n > 0\). The integer \(n\) that arises (through the Fourier analysis in equation (19)) in the description of tube waves defines the geometry of the vibrating tube. The case \(n = 0\) corresponds to a symmetric pulsation of the tube, with its axis remaining undisturbed, and is commonly referred to as the sausage wave (or mode). The case \(n = 1\) describes the kink mode, which involves lateral displacements of the tube (maintaining a circular cross-section) with the axis of the tube resembling a wriggling snake; the plasma in the neighborhood of the tube is also disturbed. There are also fluting modes \((n \geq 2)\), which leave the axis of the tube undisturbed but distort the tube boundary. See figure 4.

The restriction \(m_n > 0\) imposed on the flux tube dispersion relation means that the amplitude of a wave declines with radius \(r(>a)\), so that far from the tube there is no appreciable disturbance. Inside the tube (for \(r < a\)) the disturbance is oscillatory if \(n_0^2 > 0\) or nonoscillatory (evanescent) if \(n_0^2 < 0\). Modes that inside the tube are oscillatory in \(r\) are called body waves, because they disturb the whole of the interior of a tube. Waves that have an evanescent form inside and outside the tube are called surface waves, since they are mainly confined to the region near the boundary of the tube. See figure 5. Magnetoacoustic surface waves may also exist on a simple planar interface between two magnetized plasmas. Finally, we note that just as there are both slow and fast magnetoacoustic waves in a uniform medium, there are slow and fast tube waves, and these may be body or surface modes of the sausage, kink or fluting form. Moreover, the presence of the tube radius \(a\) in the dispersion relation means that, unlike magnetoacoustic waves in a uniform unbounded medium (equation (12)), the phase speed \(c = (\omega/k)\) of a tube wave depends on its wavelength \((2\pi/k)\): tube waves are dispersive.

Two speeds prove to be of special significance in magnetic flux tubes: the slow speed \(c_t\) of the tube and a mean Alfvén speed \(c_k\), defined by

\[
c_k^2 = \frac{\rho_0v_A^2 + \rho_ev_{ Ae}^2}{\rho_0 + \rho_e}.
\]

The speeds \(c_t\) and \(c_k\) are important in both isolated
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and magnetically embedded (photospheric and coronal) tubes. The speed \( c_s \) is called the kink speed, and it is the speed with which the kink mode of oscillation propagates for waves that are much longer than a tube radius \( k a \ll 1 \). The speed \( c_t \) is associated with both slow body and slow surface waves. The slow surface wave is of particular interest in isolated flux tubes. We may illustrate these speeds for a strongly magnetized plasma and for a photospheric flux tube. In a strongly magnetized plasma such as the solar corona (for which Alfvén speeds are much larger than sound speeds), the slow speed is close to the sound speed inside the tube. For a high-density \( \rho_0 \gg \rho \) strongly magnetized plasma (with \( B_0 \approx B \)) the kink speed is \( \sqrt{2} v_A \), and so is some 40% higher than the tube’s Alfvén speed. By contrast, in a photospheric tube with \( \rho_0 \approx \rho / 2 \) and \( v_A \approx c_t \), the tube speed \( c_t \) is about \( v_A / \sqrt{2} \) and the kink speed \( c_k \) is about \( v_A / \sqrt{3} \), so both speeds are well below the Alfvén speed.

**Ducted waves**

For a strongly magnetized plasma the dispersion relation (26) possesses two sets of modes, namely fast and slow body waves. There are no surface \( (n^2 < 0) \) waves. Both sets of modes are dispersive, with the fast waves being strongly dispersive. An interesting aspect of these waves is that the fast body waves occur only if \( v_{BA} > v_A \). Thus fast body waves occur in regions of low Alfvén speed which typically correspond to regions of high plasma density; this is the usual circumstance in coronal loops. Regions of low Alfvén speed in strongly magnetized plasmas provide wave guides for fast magnetoacoustic waves. It is interesting to report that fast kink mode oscillations in coronal loops have been identified by the TRACE spacecraft.

There are close analogies here between the behavior of fast magnetoacoustic body modes in a strongly magnetized plasma and Love waves in the Earth’s crust and Pekeris sound waves in an internal ocean layer; consequently, fast sausage waves are sometimes referred to as magnetic Pekeris waves and fast kink waves as magnetic Love waves. The distinctive dispersive wave signature they produce when impulsively excited has received interest because of its possible importance in the interpretation of oscillations detected in the corona at radio wavelengths. In particular, an impulsively generated fast body wave produces a signature which consists of three parts: a low-amplitude periodic phase, followed by a larger amplitude quasi-periodic phase, and finally a decay phase. The shortest timescales in the motions are those in the quasi-periodic phase. An estimate of the timescale \( \tau \) in the periodic phase is provided by

\[
\tau = \frac{2\pi a}{\phi v_A}
\]

where \( \phi \) (≈ 2.40) denotes the first zero of the Bessel function \( J_0 \). For a tube of radius \( a = 10^7 \) km and Alfvén speed \( v_A = 10^3 \) km s\(^{-1}\) this produces a timescale of about 1 s.

**Nonlinear waves**

Magnetohydrodynamic waves may become nonlinear, having amplitudes that are too large to permit a linear treatment. Many of the MHD waves detected in the solar wind and waves generated by a solar flare—and seen to rapidly propagate out across the solar disk—are nonlinear. Alfvén waves provide an exact solution of the nonlinear MHD equations. Nonlinearity frequently leads to wave amplitude growth and steepening, just as an ocean wave undergoes such changes on approaching the shore line. Shock waves are nonlinear waves with profiles that are particularly sharp—discontinuities in the ideal case—and across which dissipative processes act to balance nonlinearity. Conservation relations (total energy, mass, momentum, etc) determine the character of the shock wave. In keeping with the existence of both slow and fast magnetoacoustic waves, both slow shocks and fast shocks may occur. Slow shocks may arise in photospheric flux tubes and in the solar wind, and are also invoked in MAGNETIC RECONNECTION theories; fast shocks would seem to be implicated in observed blast waves generated by a solar flare. When dispersive effects are important, as in magnetic flux tubes, solitons may occur; solitons are waves that propagate nonlinearly preserving their basic shape, even after interactions with other nonlinear waves, and achieving a balance between nonlinearity and dispersion. There are theoretical grounds for believing that solitons may occur in the Sun (see, for example, SOLAR PHOTOSPHERIC MAGNETIC FLUX TUBES: THEORY).

**Bibliography**


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