Resonant Absorption

A 2D Simulation

Alexander Russell

School of Mathematics and Statistics
University of St. Andrews

1st September 2008

MHD Waves

MHD (magneto-hydro-dynamics) describes a plasma as a fluid threaded by a magnetic field.

If gravity and pressure gradients are neglected for magnetic forces, isolate 2 wave modes:

- Fast wave
  - Restoring force is magnetic pressure gradients.
  - Travels isotropically.
- Alfvén wave
  - Restoring force is magnetic tension force.
  - Energy transfer is along field lines.
  - Analogous to 'wave on a string'.

Principal of Resonant Absorption

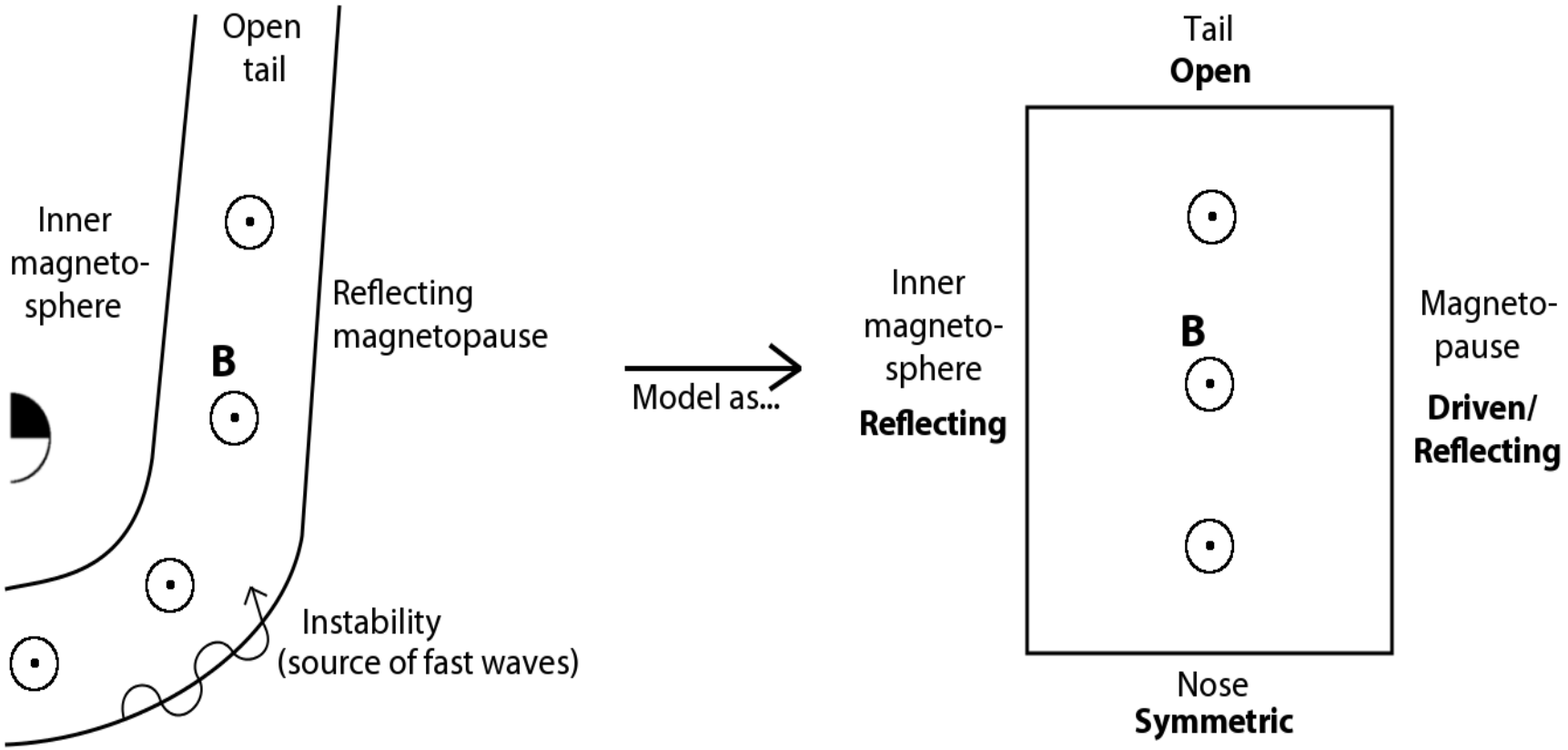
- A magnetic field line with fixed endpoints supports standing Alfvén waves.
- Let ω_A denote the frequency of the fundamental standing mode.
- A fast wave of frequency ω makes a field line resonate if $\omega = \omega_A$: fast wave energy is absorbed as the standing Alfvén wave is excited.
- ω_A is determined by field line length and the profile of v_A along the field line, where,

$$v_A(\vec{r}) = \frac{B(\vec{r})}{\sqrt{\mu_0 \rho(\vec{r})}}.$$

Hence, ω_A generally varies between neighbouring field lines.

2D Numerical Experiment - Setup

- Straight field lines of fixed length, with v_A constant on each field line, reduces problem to 2D.
- Key feature: ω_A varies in 2D, perpendicular to field lines.
- Driving and boundary conditions are based on Earth's magnetosphere.



2D Numerical Experiment - Results

Colour $\equiv$ Energy Density
(Magnetic field perpendicular to screen)

Conclusion and Remarks

Conclusion

- Resonant absorption is robust in 2D.

Remarks

- In ideal linear theory, energy density of Alfvén waves is ever increasing and phase mixing produces ever smaller length scales.
- At some point, non-linear, dissipative and/or kinetic effects will take hold.
- Robustness of resonant absorption confirmed analytically by existence of singular solution to the governing equations.
- Some features in 2D (e.g. nodes) suggest possibility of new diagnostics for Earth's magnetosphere.