Field Line Resonance with 2D Variation of Field Line Eigenfrequencies

Alexander J. B. Russell   Andrew N. Wright

School of Mathematics and Statistics
University of St. Andrews

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MHD Waves

Dynamic, large scale behaviour of the magnetosphere is naturally described in terms of MHD waves.

If gravity and gas pressure forces are neglected for magnetic forces, isolate 2 wave modes:

- Fast wave
  - Driven by magnetic pressure (field aligned magnetic field perturbation).
  - Travels isotropically.
- Alfvén wave
  - Restoring force is magnetic tension force (akin to wave on a string).
  - Energy transfer is along field lines.

These waves can couple through field line resonance.

Field Line Resonance

- A closed field line supports standing Alfvén waves with a family of discrete eigenfrequencies.
- Resonant matching of fast wave frequency to a field line eigenfrequency excites the corresponding Alfvén wave.
- This is called 'field line resonance' ('resonant absorption' in solar literature).
- Example: Excitation of Ultra-Low Frequency (mHz) Pc5 pulsations.

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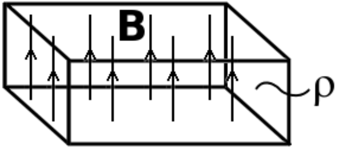
Want to show: Resonant Alfvén wave captures spatial form of fast wave.

Model

- Straight uniform magnetic field, $\mathbf{B} = B\hat{\mathbf{e}}_\mathbf{z}$, with fixed endpoints; neglect gas pressure.
  - Gives force balance in equilibrium.



- Linear, ideal MHD:
  - Valid for early times before large perturbations and small lengthscales appear.
- Density varies in 2D across field lines $\rho(x, y)$:
  - Provides 2D variation of field line eigenfrequencies.
  - Density constant along each field line allows normal mode analysis with fourier modes.
  - Consider one 'excitable' frequency for each field line:

$$\omega_A(x, y) = \frac{k_z B}{\sqrt{\mu_0 \rho(x, y)}}.$$

Analytic Solution for Asymptotic State ($t \to \infty$)

- Assume $\exp(-i\omega t)$ time dependence for perturbations at late times.
- Change to orthogonal coordinates (X, Y, z), such that $\omega_A = \omega_A(X)$ and $\omega_A(0) = \omega$.
- Solution is obtained as a generalised Frobenius series.

Analytic Solution for Asymptotic State ($t \to \infty$)

At the resonance ($X \to 0$):

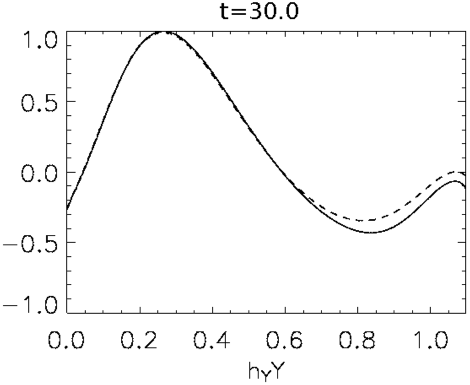
- b_z (magnetic pressure, fast wave) is finite, but velocities are singular.
- $u_Y >> u_X$: velocity perturbation is parallel to resonant surface.

$$u_Y \quad \to \quad i\left(\frac{B}{\omega[\partial\rho/\partial X]_{X=0}}\right)\left[-\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}\right]_{X=0} X^{-1}$$

- Resonance affected by B, ω and $[\partial\rho/\partial X]_{X=0}$.
- Amplitude variations of the Alfvén wave capture the spatial form of the fast wave.

Time-Dependent Velocity and Magnetic Pressure

Not only does u_Y capture $-\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$ for the asymptotic solution, it also does well in the time-dependent problem.



Solid curve: u_Y
Dashed curve: $-\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$

Intuition

- Over every period, field lines get 'a little push' from the magnetic pressure force in the resonant surface,

$$\left[-\nabla\left(\frac{\mathbf{B}^2}{2\mu_0}\right)\right].\hat{\mathbf{e}}_Y \;\sim\; -\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$$

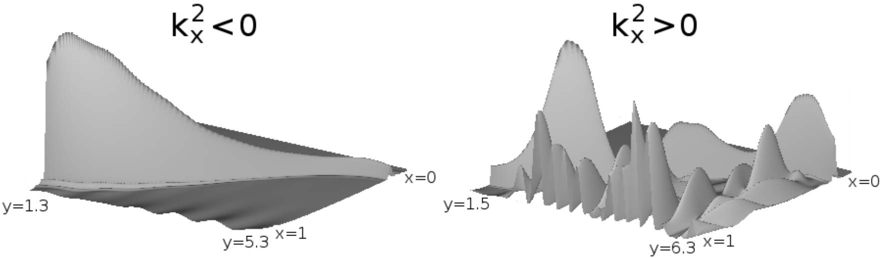
- Where resonant field lines get a large push, this builds up larger amplitude Alfvén wave than a small push.
- Hence, the amplitude of the resonant Alfvén wave captures the spatial form of the fast wave.

- Alternatively: Most energy can be passed to the Alfvén wave where the fast wave carries most energy.

Time-Dependent Energy Density

2 examples:

1. Fast wave is evanescent along resonant contour.
2. Fast wave has nodes and anti-nodes along resonant contour.



Surface plots of energy density reveal these features in the Alfvén wave.

Conclusions

We have investigated field line resonance with a 2D variation of field line eigenfrequencies.

- Field line resonance is robust.
- Amplitude variations of the resonant Alfvén wave capture the spatial form of the fast wave.
  - See this in energy density.
  - Stark correlation between u_Y and magnetic pressure force $\sim -\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$.

Russell & Wright 2009 (in preparation).