Resonant Absorption with 2D Variation of Field Line Eigenfrequencies

Alexander J. B. Russell   Andrew N. Wright

School of Mathematics and Statistics
University of St. Andrews

7th April 2009

Outline

1. Brief Introduction to Resonant Absorption
2. Model
3. Analytic Solution for Asymptotic State ($t \to \infty$)
4. Numerical Solution of Time-Dependent Problem
5. Conclusions

Introduction

- Under special circumstances (e.g. uniform medium, slab or cylinder) wave modes decouple.
- Consideration of decoupled wave modes has proven very useful, but does not tell the full story.
- 'Resonant absorption' (a.k.a. 'field line resonance') is concerned with coupling of fast and Alfvén waves.
- Resonant absorption has implications for:
  - Transportation and dissipation of wave energy;
  - Coronal seismology;
  - Excitation of ULF pulsations in the magnetosphere.

Principles of Resonant Absorption

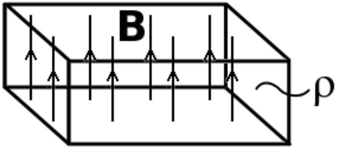
Consider field lines with a boundary condition (e.g. fixed endpoints) so that each field line has a discrete family of eigenfrequencies.

Fast wave energy is deposited as an Alfvén wave in the vicinity of a resonant surface if (and only if) three conditions are met:

1. Resonance Condition
   There is a surface at which a field line eigenfrequency matches the fast wave frequency.
2. Coupling Condition
   The fast wave is not invariant along the surface.
3. Non-Orthogonality
   The form of the fast wave along the field line overlaps the resonant eigenfunction.

Model

- Straight uniform magnetic field, $\mathbf{B} = B\hat{\mathbf{e}}_\mathbf{z}$, with fixed endpoints, threading a cold plasma:
  - Gives force balance in equilibrium and excludes slow wave.



- Linear, ideal MHD:
  - Valid for early times before large perturbations and small lengthscales appear.
- Density varies in 2D across field lines $\rho(x, y)$:
  - Provides 2D variation of field line eigenfrequencies.
  - Density constant along each field line means eigenfunctions are fourier modes.
  - Driving with a fourier mode excites a unique eigenfunction, i.e. each field line has one 'excitable' frequency,

$$\omega_A(x, y) = \frac{k_z B}{\mu_0 \rho(x, y)}.$$

Analytic Solution for Asymptotic State ($t \to \infty$)

- The analytic solution for the asymptotic state is obtained by assuming a time dependence $\exp(-i\omega t)$.
- Change to orthogonal coordinates (X, Y, z), such that $\omega_A = \omega_A(X)$ and $\omega_A(0) = \omega$.
- The solution is obtained as a generalised Frobenius series.

Leading Order Solution for Resonant Case

$$
\begin{aligned}
b_z &= \alpha_0 + \alpha_2 X^2 + O(X^3) \\
&\quad + \beta_2 \ln(X) X^2 + \ln(X) O(X^3),
\end{aligned}
$$

$$
\begin{aligned}
u_Y &= -(iB/h_{Y0}\omega[\partial\rho/\partial X]_{X=0})\alpha_0' X^{-1} + O(X^0) \\
&\quad -(iB/h_{Y0}\omega[\partial\rho/\partial X]_{X=0})\beta_2' \ln(X) X^1 + \ln(X) O(X^2),
\end{aligned}
$$

$$
\begin{aligned}
u_X &= -(iB/h_{X0}\omega[\partial\rho/\partial X]_{X=0})(2\alpha_2 + \beta_2) X^0 + O(X^1) \\
&\quad -(2iB/h_{X0}\omega[\partial\rho/\partial X]_{X=0})\beta_2 \ln(X) X^0 + \ln(X) O(X^1).
\end{aligned}
$$

- Velocities are singular at $X = 0$, but b_z (magnetic pressure, fast wave) is finite.
- $u_Y >> u_X$: velocity perturbation is parallel to resonant surface.
- This is a singular Alfvén wave.

Leading Order Solution for Resonant Case: $X \to 0$

$$b_z \to \alpha_0(Y),$$

$$u_Y \to -i\left(\frac{B}{\omega[\partial\rho/\partial X]_{X=0}}\right)\frac{1}{h_{Y0}}\frac{d\alpha_0}{dY}X^{-1},$$

$$u_X \to -i\left(\frac{B}{\omega[\partial\rho/\partial X]_{X=0}}\right)\frac{1}{h_{Y0}^3}\left[\begin{array}{c}(h_{Y0}h'_{X0}+h_{X0}h'_{Y0})\frac{d}{dY}\\ +h_{X0}h_{Y0}\frac{d^2}{dY^2}\end{array}\right]\alpha_0\ln(X).$$

- The parameter $\alpha_0(Y)$ gives the value of b_z on the resonant contour.
- The spatial form of the fast wave is imprinted on the Alfvén wave.
- Resonance affected by B, ω and $[\partial\rho/\partial X]_{X=0}$.

Excitation of Resonant Alfvén Wave

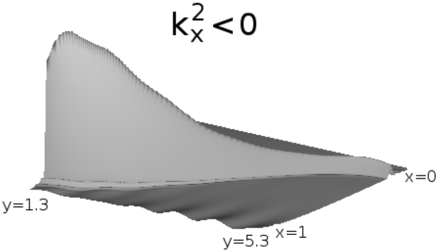
Colour $\equiv$ Energy Density
Magnetic field perpendicular to screen.

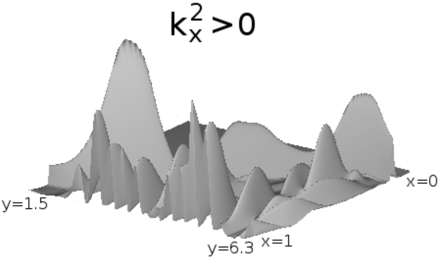
Imprinting

Spatial Form of Fast Wave Imprinted on Resonant Alfvén Wave

We present 2 examples:

1. Fast wave is evanescent along resonant contour.
2. Fast wave has nodes and anti-nodes along resonant contour.



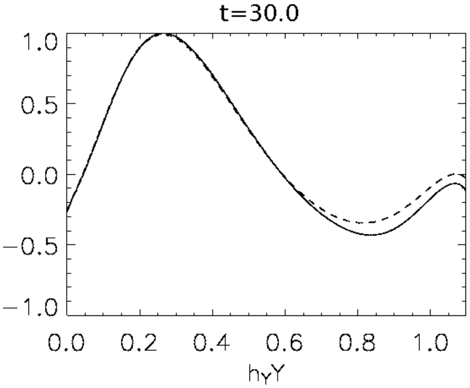


Surface plots of energy density reveal these features in the Alfvén wave.

Imprinting

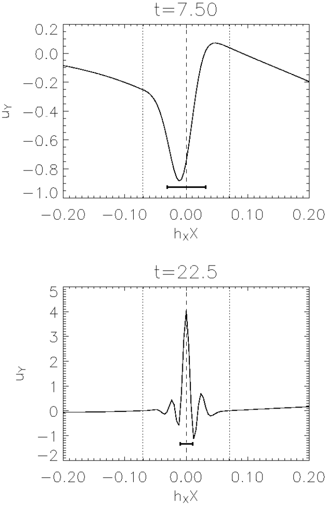
Spatial Form of Fast Wave Imprinted on Resonant Alfvén Wave

Not only does u_Y capture $\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$ for the asymptotic solution, it also does well in the time-dependent problem.



Solid curve: u_Y
Dashed curve: $-\frac{1}{h_Y}\frac{\partial b_z}{\partial Y}$

Phase Mixing



$$L_{ph}(t) \;=\; 2\pi(t|\nabla\omega_{\mathbf{A}}|)^{-1}$$

Conclusions

When field line eigenfrequencies vary in 2D:

- Resonant absorption is robust.
- The spatial form of the fast wave is imprinted on the Alfvén wave.
- The resonant Alfvén wave phase mixes with smallest scales given by the generalised phase mixing length.
- Techniques used to solve the asymptotic problem can be combined with previous works (Thompson & Wright 1993) to solve a 3D density profile. Tirry & Goossens 1995 can guide generalization to warm, resistive plasmas and poloidal field.

Russell & Wright 2009 (in preparation).